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NUMERICAL AND THEORETICAL MODELING OF HOLLOW CYLINDERS FOR SAND PRODUCTION

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DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of the University of Nicosia. The work presented in this thesis has been composed solely by myself and has not been previously submitted, in whole or in part, to this or any other institution for a degree, diploma or other qualifications.

Panos Kakonitis. Date :05/05/2026



ABSTRACT

This doctoral research establishes a predictive framework for volumetric sand production by investigating the coupled hydro-mechanical processes of plastic yielding, material degradation, and hydrodynamic erosion in weakly consolidated formations. The investigation integrates two complementary approaches. First, a fully coupled, poro-elasto-plastic finite element model utilizing an Arbitrary Lagrangian-Eulerian (ALE) formulation to analyze stress evolution and material failure in hollow cylinder tests and secondly, a novel semi-analytical framework that reduces the erosion dynamics to a system of coupled ordinary differential equations governed by mean porosity and plastic zone depth.

The primary contribution of this work is the quantitative demonstration that the sand production coefficient (λ), a key parameter in hydrodynamic erosion models, exhibits a strong power-law dependence on the applied far-field stress. This relationship, first identified through numerical simulations, is incorporated into the semi-analytical model as a constitutive law for material degradation. The resulting kinematic formulation accurately predicts cumulative sand production, plastic zone evolution, and porosity changes, showing excellent agreement with experimental data from multiple hollow cylinder tests under variable stress and intermittent flow conditions.

Collectively, this research advances the state-of-the-art by providing a mechanistically grounded and computationally efficient methodology for calibrating erosion parameters and predicting sand production. The validated models provide a robust foundation for enhancing proactive sand control strategies and optimizing well completion designs in challenging geomechanical environments.

Keywords: Sand production, Sanding criterion, Surface erosion, Hydrodynamic models, Sand production coefficient, Degradation, Poroelastoplasticity, Hollow cylinder test.

DEDICATION

To my family and friends, with gratitude and love.

May the pursuit of knowledge remain an endless journey, a flame we tend, share, and carry forward together.

As the desert reminds us, *“We are all sparks, learning to shine in the vastness of life”*.



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NOMENCLATURE

Greek symbols

λ sand production coefficient

Λ plastic zone depth

ϕ porosity field

ϕ_0 initial porosity

$\bar{\phi}$ mean porosity within the plastic zone

Φ friction angle of the rock

Φ_0 initial friction angle

σ_r radial stress

σ_θ tangential stress

σ_{out} external radial stress

ρ_s mass density of the rock

Latin symbols

a_D rate of softening

a_H exponent in hydrodynamic constitutive

C cohesion of the rock

C_0 initial cohesion

H height of the hollow cylinder

q Darcy flux

Q volumetric flow rate

r radial distance from the cylinder axis

r_{in} inner radius of the hollow cylinder

r_{out} outer radius of the hollow cylinder

S_0 uniaxial compressive strength

u speed of advancement of plastic yielding

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CHAPTER I - INTRODUCTION

1.1 Preamble

This chapter discusses the core aspects of sand erosion phenomena encountered in hydrocarbon recovery and describes how laboratory-scale investigations can advance our understanding of this issue. Key sections address the problem definition, the limitations between lab and field conditions, the main hypotheses guiding the work, the methods used, and the novel contributions that emerge from the analysis.

1.2 Problem statement

In the context of hydrocarbon recovery, particularly within oil and gas extraction, *sand erosion* presents a critical and complex challenge. It refers to the abrasive degradation of wellbore components and surface equipment caused by the entrainment of sand particles with the produced fluids, such as oil, gas, and water. The erosion mechanism is fairly well understood. The communication between the well and the reservoir rock is achieved either by perforations or open-holes which both are stressed with the compressive in nature in-situ stresses as shown in figure 1.

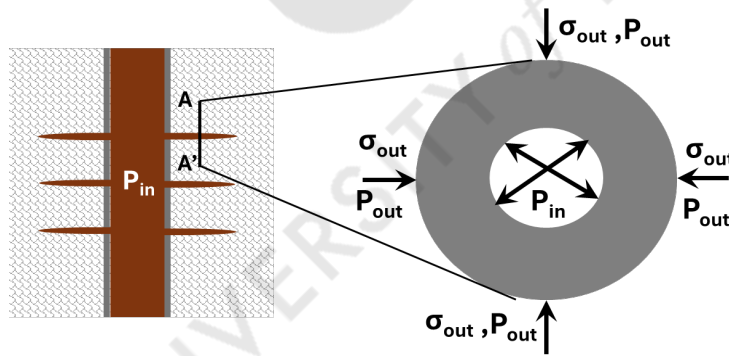


Figure 1.1 Schematic representation of the wellbore completion and the reduced geometry for mathematical analysis.

These openings are also stressed by the drawdown pressure that is responsible for producing fluids. Sand production starts when the insitu stresses cause local failure of the openings and drag forces from the drawdown pressure cause the locally failed grains to separate from the rock and enter the production system. The mobilized particles can fill or block the wellbore, plug perforation channels and flowlines, collapse structural components, and damage both downhole and surface equipment, often requiring expensive remedial actions and workover operations. This synergetic effect is fully coupled and reiterates until

catastrophic failure occurs. If not effectively managed, this phenomenon can compromise the structural integrity of vital systems, reduce operational efficiency, and significantly shorten the service life of production infrastructure (Li et al., 2018; Rahmati et al., 2013).

Furthermore, Mahmud et al., (2020) emphasized that the financial impact of sand erosion is far from negligible. Industry reports estimate that erosion-related issues cost producers hundreds of millions of dollars annually. Therefore, any detailed investigations into sand erosion physics that will help understand the underlying mechanisms involved are valuable contributions not only for academia but for the industry as well. Understanding the conditions under which sand production and transport occur enables the development of better prediction, prevention, and mitigation strategies. These are crucial for sustaining reservoir productivity, maintaining equipment performance, and minimizing operational and economic risks. (Mahmud, et al., 2020)

1.3 Limitations Between Labs and Field

Laboratory research, while essential for controlled experimentation, presents inherent limitations when compared to field conditions. One major constraint is rock core sample scale and boundary conditions. Laboratory tests are typically conducted on small, often idealized samples that fail to reflect the full complexity of natural subsurface formations. These tests cannot adequately capture the range of stress regimes, rock heterogeneities, and fluid flow pressure dynamics present in large-scale reservoirs. Geological features such as layering, natural fractures, formation damage, and variations in in-situ stress are difficult to replicate accurately in any lab setting with any experimental apparatus. Additionally, fluid flow and erosion mechanics differ significantly between lab and field conditions. Field operations involve longer production times and more intricate fluid-formation interactions, which can lead to time-dependent effects such as fines migration, permeability changes, and produced sand accumulation, phenomena that may be underestimated or missed entirely in shorter-duration lab tests. Another important factor is instrumentation and data interpretation. In the field, downhole measurements are often limited by the constraints of sensor technology, and spatial coverage, resulting in data being collected from fewer points and at coarser resolution. In contrast, laboratory instruments can capture variables such as local stress, strain, and pore pressure with high precision and temporal resolution. However, applying laboratory results to real field conditions must be done with caution. Differences in scale, natural variability in the field, and changes in stress distribution can make direct comparisons misleading if these factors are not carefully considered.

Nevertheless, laboratory experiments will always play a crucial role in advancing the understanding of geomechanical systems by offering a controlled setting to test hypotheses, explore fundamental mechanisms, and calibrate predictive models. These controlled conditions enable the isolation of specific

variables and the study of their effects in detail, which can be difficult to achieve in the complexity of field environments. When laboratory findings are thoughtfully integrated with field observations, numerical simulations and mathematical modelling they enhance the accuracy and reliability of geomechanical predictions.

1.4 Hypotheses

For the purposes of this study, we have formed two main hypotheses to investigate and are as follows:

- (i) In sand production tests, that are performed in hollow cylinders in the laboratory, the fluid pressure of the permeating fluid has significantly lower in magnitude than confining stress magnitude. This hypothesis states that the confining stress acting at the external boundaries of the hollow cylinder to simulate lithostatic-confining stresses is considerably higher than the fluid pressure generated from a servo pump to create the necessary drawdown pressure simulating production. This first observation highlights the dominant role of mechanical stress in controlling sand erosion stability.
- (ii) Hydrodynamic models for predicting erosion involve constitutive law or sanding criteria that originate from filtration theories. These models present the so-called sand production coefficient (λ) which is a constitutive model parameter with dimensions of inverse length and is usually calibrated by sand erosion tests. This parameter is thought to characterize the strength of erosion in the constitutive law, but its nature and its dependencies have not been clarified to date hence we attempt to present insights in this key parameter.

1.5 Scope, Objectives, and Methods Used

This research work aims to investigate (i) the effects of fluid pressure gradients that are generated in the experimental test of the hollow cylinder test that is routinely used in petrophysics laboratories for quantifying sand production. Furthermore, it also seeks to (ii) shed some light on the physics of the erosion coefficient (λ) that arises from filtration theories and is found in hydromechanical erosion laws for modelling and predicting sand production. The methods used are categorized into two main approaches:

- (i) Numerical modelling with fully coupled finite elements. The simulations are constructed for the hollow cylinder test at lab scale with the intention to understand how the sand production coefficient (λ) affects the on-set and rate of sand detachment in hollow cylinder experiments. Computational simulations are used as a back analysis tool to compare with the experimental curves of Papamichos et al., (2001). With this way the only free d.o.f is the sand production coefficient (λ) under different confining stresses. The end result is the construction of an empirical formula to relate the sand production coefficient (λ) as a function of the confining stresses benchmarked with experimental

tests. For further comparisons the finite element model was compared with the analytical solutions of Gravanis et al., (2015) showcasing that the finite element model can be easily extended to account for complex boundary conditions and nonlinear material behavior. Also, possibly extend to field-scale offering a more realistic prediction of erosion in actual wells.

- (ii) Mathematical modelling by developing semi-analytical hydro-mechanical framework for modeling the lab-scale hollow cylinder test. The mathematical method is based on the kinematic formulation of the hydro-mechanical models of Vardoulakis et al. (1996) and Papamichos et al. (2001). The mathematical models incorporate stress-dependent and rate-dependent erosion terms to predict the onset of sand production. These models aim to establish the threshold conditions for failure and erosion under varying operational parameters. The end result was the construction of a simple and useful tool that permits quick estimates of sand production. Furthermore, it can be used to study efficiently different constitutive erosional laws while respecting the physics of mechanics, hydrodynamics and degradation of the rock. We then use the newly proposed mathematical framework to study further the sand production coefficient (λ). It is important to note that the mathematical semi-analytical framework is restricted to lab-scale where pressure drawdown is small compared to the confining stress but can be extended to incorporate effects of high-pressure drawdowns encountered in actual wells. We found that the sand production coefficient can also exhibit a decreasing exponential dependence extending our previous contribution by deriving a new empirical formula. For completeness, the mathematical model was also applied to different sanding experiments with varying external stress and flow rate exhibiting good agreement with the laboratory datasets.

1.6 Novelty and Contributions

A) Insights into the Strength of Erosion. A primary contribution of this research is the establishment of a novel empirical relationship for the sand production coefficient (λ), a critical parameter governing hydrodynamic erosion. Through the analysis of laboratory-scale hollow cylinder tests, it was demonstrated that the initial transient erosion phase is of limited duration, justifying the application of a steady-state approximation to model the primary erosion process. This critical insight enabled the derivation of a first of its kind empirical power law relationship that explicitly links the magnitude of λ to the applied confining stress. This finding provides insights for the constant λ thereby improving the predictive hydro-mechanical modeling capability of simulators. (Kakonitis et al., 2023a).

B) New Analytical Hydro-Mechanical Model for Sand Production. We introduced a novel semi-analytical framework for predicting sand production dynamics in hollow cylinder tests. The central innovation is a kinematic formulation that reduces the complex, spatially distributed hydro-mechanical problem to a

tractable system of two coupled ordinary differential equations (ODEs). These equations govern the temporal evolution of the system's two principal state variables. (i) the mean porosity of the yielded rock and (ii) the depth of the plastic zone. This reduction in complexity offers a computationally efficient alternative to fully numerical methods like finite element analysis, while preserving the essential physics of the erosion process. The framework is specifically designed to handle variable boundary conditions, such as intermittent fluid flow and step-wise increases in confining stress, which are typical of laboratory experiments. By providing direct analytical relationships between state variables, the model offers clear physical insight into material degradation and serves as a powerful tool for the rapid calibration of erosion parameters from experimental data. (Kakonitis et al., 2023b)

C) New proposed erosion law. The culmination of the research is the enhancement and validation of the semi-analytical hydro-mechanical framework by integrating a physically-grounded, stress-dependent erosion law. We have improved the initial framework, that relied on the simplifying assumption of a constant sand production coefficient (λ) of Kakonitis et al., (2023a; 2023b). This work advances the model by replacing that assumption with the empirical power-law relationship by linking λ to the applied confining stress that was established in the first phase of this research. The advantage of the enhanced formulation was demonstrated through a direct comparative analysis of the two approaches. By modeling the same experimental scenario using both (i) the constant- λ and (ii) the power-law λ assumptions, it was shown that the stress-dependent formulation provides a more accurate prediction of cumulative sand production. The refined model was validated against experimental results showing a marked improvement in predictive accuracy over the constant- λ version, particularly during the later stages of the experiment characterized by higher confining stresses. This process not only validates the central hypothesis of the thesis but also provides a clear methodology for quantifying the impact of different constitutive erosion laws on sand production predictions. (Kakonitis et al., 2024).

1.7 Research Organization

The research work is organized in the following way. In chapter 1 we present the literature review that has been performed, providing the necessary background and context for this study. Chapter 2 discusses the stress dependency of the sand production coefficient within the framework of hydro-dynamical sanding criteria utilizing fully coupled finite elements. The numerical model is tested against experimental data and analytical solutions from the literature. The core of this work is presented in chapter 3 where we present the semi-analytical hydro-mechanical framework for hollow cylinder sanding tests and demonstrate its applicability on various experiments from literature. We also present the extension of the semi-analytical framework to handle variable

confining stress and flow rate replicating even better the experimental conditions. Finally, in chapter 4 we summarize the key findings and suggest future extensions of this work.



CHAPTER 2 – ON THE MECHANICS OF SAND PRODUCTION

2.1 Preamble

This chapter offers a thorough discussion on the mechanics of sand erosion analyzing the phenomena that are common in hydrocarbon recovery operations. This chapter is largely based on review of existing literature, and we attempt to explain how laboratory-scale investigations can greatly improve our understanding of the physics involved in sand production. The following sections are organized in such way to address the main elements of this research work. These include: (i) a clear definition of the sand erosion problem in the oil and gas industry, (ii) a critical analysis of the inherent limitations and discrepancies between controlled laboratory conditions and complex, real-world field environments, (iii) an outline of the main assumptions of this work, (iv) a description of the methods and mathematical setups used to perform this research and finally, (v) a summary of the novel contributions and key insights that emerge from the subsequent analysis. Through this structured approach, the chapter seeks to bridge the gap between theoretical understanding and practical application, offering a solid foundation for sand erosion predictions.

2.2 The mechanism of sand particle production from erosion processes

In a broad mechanical context, sand erosion is the process whereby solid sand particles fail from the parent rock and in turn are transported by a fluid medium such as water, oil, or gas from the face of the completion towards the well and are produced along with the hydrocarbons (Skjærstein et al., 1997; Abduljabbar et al., 2022).

In turn, the material gap that was created from the erosion process at the completion face, creates stress concentration and further material failure that drag from the produced fluids will pick up the failed grains and send them towards the well (Skjærstein et al., 1997; Wong et al., 2012). This process continues slowly in the beginning but after some time it escalates fast, delineating a dynamic character of the phenomenon that can weaken the structural integrity of vital systems, decrease operational efficiency, and reduce the lifespan of production infrastructure (Wong et al., 2012; Skjærstein et al., 1997). If left unmanaged, sand erosion may cause frequent equipment failures, production halts, and higher maintenance costs, thereby undermining both economic and safety goals leading eventually to catastrophic failure (Wong et al., 2012; Abduljabbar et al., 2022). During transportation, severe wear and abrasion of the surfaces they

contact takes place costing the industry few billions annually (Mahmud HB et al., 2020). From an environmental point of view, erosion is also a natural process of sedimentology that plays a vital role in shaping land and marine landscapes, contributing to changes in landforms and, in some instances, causing degradation, especially in coastal and arid areas (Abduljabbar et. al., 2022).

The implications of sand erosion differ depending on the environment in which it occurs. In industrial settings, sand erosion can cause severe damage to infrastructure, including wall thinning in pipelines, the formation of cracks or leaks, and eventually, equipment failure. In natural settings, it can lead to the gradual degradation of ecosystems, speed up desertification, and change hydrological patterns. These effects lead to high repair and replacement costs while also presenting significant safety risks to personnel and operations (Morita et. al., 1991, Rahmati et al., 2013, Volont'e et. al., 2013, LI et. al., 2018, Eshiet KII et. al., 2019 and Kakonitis et. al., 2023a).

A solid understanding of how sand erosion works and its effects is essential for effective and resilient hydrocarbon extraction methods.

2.2.1 Factors influencing sand erosion

Building upon the general mechanisms of sand erosion, the focus turns to the specific context of hydrocarbon recovery. In such operations, sand production and subsequent erosion are not only common but, as explained above, are also critical concerns that can compromise equipment integrity, reduce production efficiency, and elevate operational risks. Thus, understanding the factors that influence sand erosion is essential for developing targeted mitigation strategies and ensuring the long-term reliability of extraction infrastructure (Vardoulakis et. al., 1996, Papamichos et. al., 2001 and Kakonitis et. al., 2023a,b).

The primary factors governing sand production and erosion are extensively documented within the literature.

(A) The onset and severity of this phenomenon are dictated by a complex interplay of geological, reservoir, and operational conditions. Geological determinants include the intrinsic properties of the formation, such as rock type, sediment composition, and degree of consolidation (Bratli et. al., 1981, Van den Hoek et al., 2000). These are coupled with reservoir conditions, which encompass the in-situ stress state, pore pressure, temperature, and fluid properties. Finally, operational parameters, including well completion strategies, production rates, and pressure drawdown, directly influence the near-wellbore stress concentration and the hydrodynamic forces that

ultimately trigger the mobilization of sand particles (Vardoulakis et. al., 1996 and Morita et. al., 1989). The interaction of these elements controls the erosion patterns and severity throughout both subsurface and surface infrastructure (Ranjith, P. et al., 2014).

(B) Once sand production has initiated, the subsequent erosional damage is governed by a distinct set of physical and mechanical parameters. The characteristics of the transported solids, specifically their size, angularity, and hardness, dictate their abrasive potential (Vardoulakis et. at., 1996, and Kakonitis et. al., 20023a). Simultaneously, the hydrodynamic conditions of the multiphase flow, including fluid velocity and turbulence intensity, determine the kinetic energy transferred upon impact. Furthermore, the response of the production infrastructure is a function of the material properties of the impacted surfaces and the impingement angle of the particles. The interplay between these factors governs the dominant wear mechanisms and ultimately controls the rate of material degradation (Stavropoulou et. al., 1998 and Kakonitis et. al., 2023b).

(C) Reservoir conditions are integral to both the geomechanical stability of the formation and the hydrodynamic forces responsible for sand detachment (Vardoulakis et. at., 1996, Papamichos et. al., 2001 and Kakonitis et al., 2023 a,b). The in-situ stress state and temperature govern the rock's intrinsic mechanical properties, such as cohesion and compressive strength. Concurrently, the composition and saturation of reservoir fluids (oil, gas, and water) can weaken inter-granular cementation and modify particle transport mechanisms. Moreover, production dynamics, specifically the rate and method of fluid extraction, induce localized changes in the effective stress field and pressure gradients near the wellbore. These production-induced alterations are often the direct trigger for formation failure and subsequent sand production.

(D) Finally, operational parameters represent the most direct engineering control on sand production. These strategies govern the near-wellbore flow dynamics and effective stress regimes (Morita et. al., 1989, Vardoulakis et. al., 1996 and Van den Hoek et. al., 2000). Consequently, suboptimal operational choices can exacerbate formation instability, leading to increased sand production rates and accelerated erosional degradation of production facilities.

To facilitate a more systematic analysis of the numerous variables affecting sand erosion, it is helpful to synthesize the previously discussed factors into broader, interrelated categories. While the mechanisms behind sand production and erosion, as explained, are inherently complex, driven by geological formations, fluid dynamics, and operational practices, they can be effectively examined by grouping them according to their dominant physical influences. This classification

not only enhances analytical clarity but also supports the development of predictive models and mitigation strategies. Accordingly, the primary factors influencing sand erosion in hydrocarbon recovery are organized into three fundamental categories: (1) stresses and pressure-related factors, (2) temperature, and (3) rock type. These categories provide a framework for understanding the underlying causes of erosion and their implications for reservoir performance and equipment durability.

2.2.1.1. Stresses-reservoir pressure and fluid dynamics

Reservoir pressure has a critical influence on sand production, directly affecting the mechanical stability and stress equilibrium of the rock formation. Under normal conditions, in-situ stresses, comprising both overburden and pore pressure, maintain a stable configuration in which grain cohesion and rock integrity are preserved. However, when reservoir pressure declines due to fluid extraction, the balance of stresses shifts, increasing the effective stress acting on the formation. This change can induce mechanical failure, particularly in formations with low compressive strength, resulting in the grain detachment, disaggregation, and mobilization of sand particles into the wellbore. Such failure mechanisms are especially prevalent in unconsolidated or poorly cemented formations, where weak grain-to-grain bonding provides minimal resistance to deformation under stress (Bratli & Risnes, 1981, Morita et al., 1989; Vardoulakis et al., 1996).

In parallel, the rate of fluid extraction and the associated pressure drawdown play a decisive role in triggering sand production. Rapid or aggressive pressure drawdowns can create steep pressure gradients near the wellbore, enhancing the likelihood of tensile fracturing or shear failure in the surrounding rock. Enhanced recovery methods, such as water flooding and gas injection, modify the in-situ stress distribution and disrupt the pore pressure equilibrium, thereby affecting the overall stability of the reservoir formation. Additionally, the velocity and turbulent fluid flow, especially in high-rate production wells, amplify the erosive forces exerted on both the reservoir rock and production equipment. These forces are further intensified by the presence of sand-laden fluids, leading to increased wear rates in materials exposed to the flow (Vardoulakis et al., 1996, Stavropoulou et al., 1998 and Fetrati & Pak, 2020).

2.2.1.2. Thermal influence on rock-fluid interactions

Temperature is another significant factor influencing sand production, though its impact is often interlinked with both mechanical and chemical processes within the reservoir. Elevated reservoir temperatures can alter the mechanical properties of the rock matrix, reducing rock strength and modifying grain bonding, particularly in thermally sensitive formations such as those containing clay minerals. Thermal expansion and contraction resulting from the injection or production of fluids at temperatures different from the native reservoir can induce thermal stresses, which may contribute to the initiation or propagation of fractures, facilitating sand movement (Ram et. al., 2025). Moreover, temperature can influence fluid properties such as viscosity, density, and interfacial tension, which in turn affect the flow regime and the force exerted on formation sand. Thermal recovery techniques, including steam injection and cyclic steam stimulation, can enhance hydrocarbon mobility but also destabilize the formation if not carefully controlled, resulting in increased risk of sand intrusion.

2.2.1.3. Geological composition and mechanical integrity

The lithological characteristics of the reservoir rock, particularly its grain size distribution, mineral composition, cementation quality, and porosity, play a foundational role in determining its susceptibility to sand production. Unconsolidated or weakly consolidated sandstone formations, often prevalent in shallow or young geological settings, are especially prone to mechanical failure under production stresses. The morphology and mechanical properties of the grains, including their size, shape, angularity, and hardness, significantly influence the erosive impact of dislodged particles. Angular and harder grains tend to cause more severe erosion upon impact with metallic surfaces such as tubing, valves, and downhole equipment. The material properties of these surfaces, including hardness and resistance to abrasion, further influence the erosion rate (Dusseault & Santarelli, 1989; Aadnoy & Looyeh, 2011 and Ayal & Sadeq, 2024).

Furthermore, the angle of particle impingement plays a critical role in the erosion mechanism. Direct impacts often result in localized pitting and plastic deformation, while oblique impacts tend to cause surface fatigue, leading to progressive material loss over time. Rock heterogeneity and the presence of natural fractures further complicate sand production and erosion dynamics. Fractured, stratified, or heterogeneous formations can create preferential pathways for sand-laden flow, making it more difficult to predict and control erosion patterns across the

reservoir and wellbore environment (Vardoulakis et al., 1996, Papanastasiou & Zervos, 2004, Papamichos, 2010).

2.2.2 The mechanism of sand erosion in weak unconsolidated formations

Sand production in oil and gas wells is most commonly associated with unconsolidated and/or weakly consolidated sandstone formations. Still, it is essential to consider all rock types and geological conditions that may contribute to this phenomenon. Understanding the mechanisms behind sand erosion in weak formations is essential because it is closely linked to the physical and mechanical properties of the reservoir rock. The degree of cementation, grain cohesion, permeability, and structural features such as fractures all influence the formation's resistance to sand production (Issa et al., 2022 and Ranjith et al., 2014).

The following rock types are particularly prone to erosion under production conditions:

- i. Unconsolidated sandstones (see Figure 2.1) represent the most common source of sand production. They are sedimentary rocks composed mainly of sand-sized mineral particles or rock fragments that are loosely packed and poorly cemented. The absence of sufficient cementation between grains makes these formations highly susceptible to erosion under fluid flow (Fjær et al., 2008 and Subbiah et al., 2021).

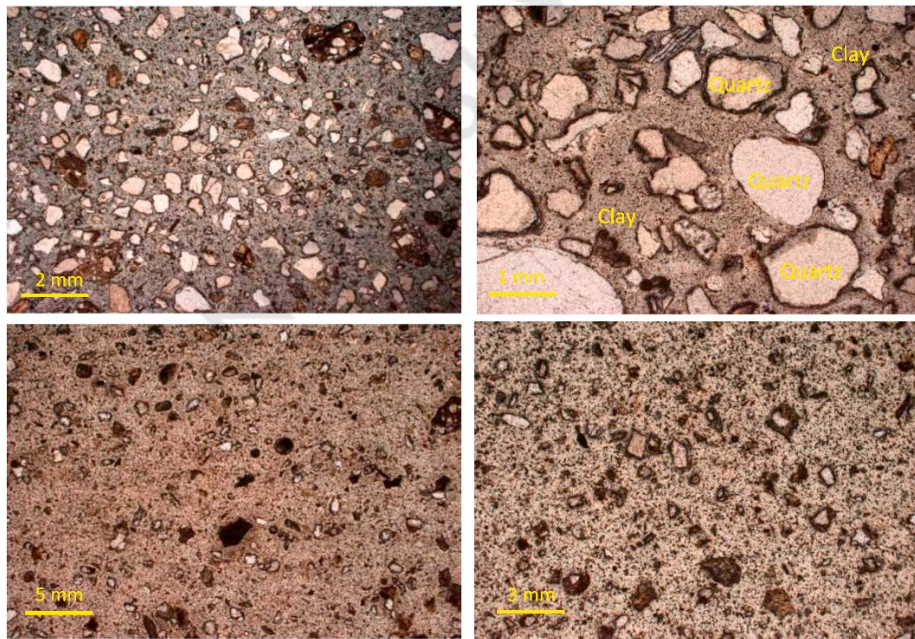


Figure 2. 1 Unconsolidated Sandstone samples on different scales (Gholami et al., 2021)

- ii. Weakly consolidated sandstones exhibit slightly more cohesion than unconsolidated sandstones but remain mechanically fragile. These share similar characteristics but possess minimal cementation. While this limited bonding provides some structural integrity, it is often insufficient to withstand the mechanical stresses induced by production fluids. As a result, even slight changes in reservoir pressure can dislodge grains, initiating sand transport. Figure 2.2 presents weak consolidated sandstones at sample and core scale (Gao et al., 2025).
- iii. Fractured rocks, regardless of lithology, can also become vulnerable to sand production when intersected by extensive or unstable fractures. These fractures offer preferential flow paths that can destabilize the formation, particularly if the rock matrix within or adjacent to the fractures is unconsolidated or weakly cemented.

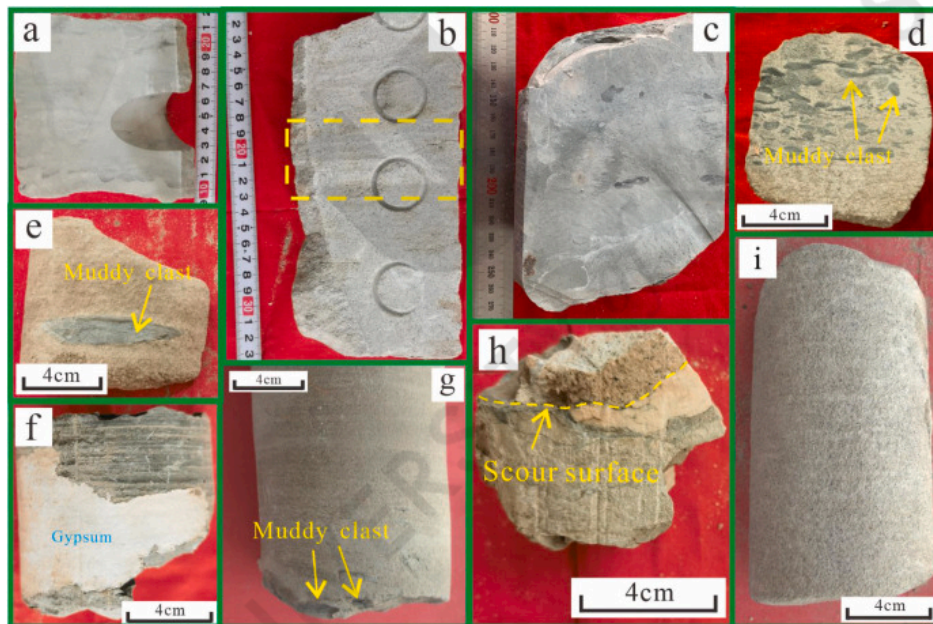


Figure 2.2 Weak consolidated Sandstone examples at sample and core scale (Gao et al., 2025)

- iv. High-permeability formations often facilitate rapid fluid flow, increasing the risk of particle mobilization. These formations further contribute to sand production risk. The ease of fluid movement through such rocks can enhance the drag forces acting on grains, especially in formations with low mechanical strength (Fjar et al., 2008).

Among the various rock types discussed, weakly unconsolidated formations present a particularly high risk for sand production. Their unique geological properties not only make them more vulnerable but also serve as a prime context for examining the fundamental mechanisms behind sand erosion. Understanding how sand is mobilized and transported under production conditions requires a closer look at the specific physical interactions that lead to grain destabilization and eventual detachment within these fragile formations. These formations are characterized by poorly cemented sand grains with minimal grain-to-grain bonding. Upon the initiation of production, reservoir fluids such as oil, gas, or water flow through the porous rock matrix, generating differential pressures and stress concentrations within the pore spaces. As the fluid navigates in the formation, it exerts drag forces on individual grains. When these forces surpass the stabilizing forces, such as gravity and cohesion, the grains become dislodged, initiating grain detachment. The loose packing of such formations significantly reduces mechanical resistance, facilitating sand mobilization (Fjar et al., 2008 and Aadnøy et al., 2011).

2.2.3 Sedimentary rock formation and its influence on mechanical strength

The tendency of sedimentary formations to undergo erosion and sand production is closely tied to the geological processes that govern their development. Each phase, from initial weathering and sediment transport to deposition and diagenesis, plays a critical role in determining the rock's consolidation level, the strength of grain-to-grain bonding, and its overall mechanical integrity.

To fully understand the mechanical behavior of sedimentary formations, it is essential to examine the key geological phases that shape their properties. The geomechanical behavior of sandstone reservoirs, particularly their susceptibility to sand production, is a direct consequence of their geological history. The journey from source rock to a weakly consolidated reservoir involves a sequence of interconnected processes, beginning with weathering and culminating in diagenesis. Each stage in this sequence imparts critical characteristics to the rock fabric that ultimately governs its mechanical integrity under production-induced stresses. The genesis of siliciclastic sediments begins with the weathering of pre-existing source rocks. This initial breakdown occurs through mechanical, chemical, and biological mechanisms.

Mechanical weathering, such as abrasion or thermal expansion, disintegrates the rock into loose grains without altering their mineralogy, thereby creating the foundational granular material for future sandstones. Concurrently, chemical weathering alters the composition of less stable

minerals, often leading to the formation of authigenic clays. The presence of these clays is of particular geomechanical significance, while they can act as a weak binding agent, they can also inhibit the later precipitation of stronger mineral cements, thereby predisposing the rock to future instability. Once liberated from the source rock, these sediments are subject to erosion and transportation by agents such as water, wind, or ice.

The transport phase is critical in determining the textural maturity of the sediment. Prolonged or high-energy transport results in increased sorting by grain size and enhanced rounding of individual particles. While this process yields texturally mature sediments with high initial porosity and permeability, which are the main desirable parameters for a hydrocarbon reservoir rock, it also creates a granular framework that possesses little to no initial cohesion. The ultimate strength of such a rock will therefore be almost entirely dependent on post-depositional cementation.

The depositional environment dictates the initial fabric and stratigraphic architecture of the potential reservoir rock. High-energy environments, such as fluvio-deltaic systems, tend to deposit well-sorted, coarse-grained sands, while low-energy environments, like deep marine settings, favor finer sediments. The nature of deposition influences the initial packing density and heterogeneity of the sediment body, which in turn controls the pathways for subsequent fluid migration and diagenetic processes.

After deposition, the accumulated sediments undergo progressive burial, which initiates the process of diagenesis which includes the sum of all physical, chemical, and biological changes that occur post-deposition. The initial stage of diagenesis is dominated by mechanical compaction, where the increasing overburden stress reduces pore volume by forcing grains into a denser packing arrangement. This process increases the number of grain-to-grain contacts and provides a degree of mechanical stability. However, this compaction-induced strength is frictional and is not a substitute for true cohesive strength, which is derived from cementation. Cementation is the most critical diagenetic process in determining the final geomechanical properties of the sandstone. During this stage, minerals such as quartz, calcite, or various clays precipitate from pore fluids at the grain contacts, binding the granular framework together. The extent, type, and distribution of this mineral cement are the primary determinants of whether a sandstone becomes a well-consolidated, competent rock or remains a weakly consolidated formation. Insufficient cementation, the precipitation of mechanically weak clay cement, or the later dissolution of

existing cements results in a rock fabric with low cohesive strength. It is precisely these weakly consolidated formations that are highly susceptible to sand production when the in-situ stress equilibrium is disturbed by drilling and hydrocarbon extraction (Araujo-Guerrero et al., 2022).

2.2.4 Failure modes in erosion mechanics

The primary experimental method for investigating the geomechanical behavior of sandstones under conditions analogous to a wellbore is the hollow cylinder test. This approach is widely adopted because its geometry inherently replicates the stress concentration and radial flow convergence that occur around a production well, providing a robust physical model for studying failure mechanisms. The specimens used in these laboratory experiments are typically sourced in two ways. The preferred material consists of reservoir core samples, which are cylindrical sections of rock extracted directly from the subsurface formation of interest during drilling operations. These samples offer the highest fidelity as they preserve the in-situ mineralogy, texture, and diagenetic history of the reservoir. When reservoir core is unavailable or prohibitively expensive, an alternative is to use outcrop analogues. These are rock samples collected from surface exposures of geological formations that are known to be stratigraphically and positionally equivalent to the target reservoir. While they may exhibit some degree of surface weathering, carefully selected outcrop samples can serve as effective proxies for evaluating the fundamental mechanical properties of the reservoir rock (Fjær et al., 2008, Aadnoy and Looyeh 2011 and Gholami et al., 2016).

These experimental studies have demonstrated that cavity failure, the breakdown of the rock surrounding a central hollow, occurs in three distinct patterns depending on the mechanical characteristics of the sandstone. These observations have led to the development of a classification system for sandstones based on their failure response. According to Papamichos et al., (2008) sandstones are grouped into three categories: Class A (brittle), Class B (ductile), and Class C (compactive). This classification reflects the rock's ability to deform and fracture under stress and is directly linked to the observed failure modes during laboratory testing.

Measurements of sand production from these experiments reveal a clear relationship between the type of failure and the rate at which sand is released. Class A sandstones exhibit a rapid increase in sand production as stress intensifies. In contrast, Class B sandstones initially

produce sand at a measurable rate but eventually show a deceleration in production. Class C sandstones exhibit a more gradual and continuous increase in sand content with increasing stress.

The failure morphology observed in hollow cylinder tests provides a direct visualization of the rock's constitutive response is illustrated in Figure 2.3.

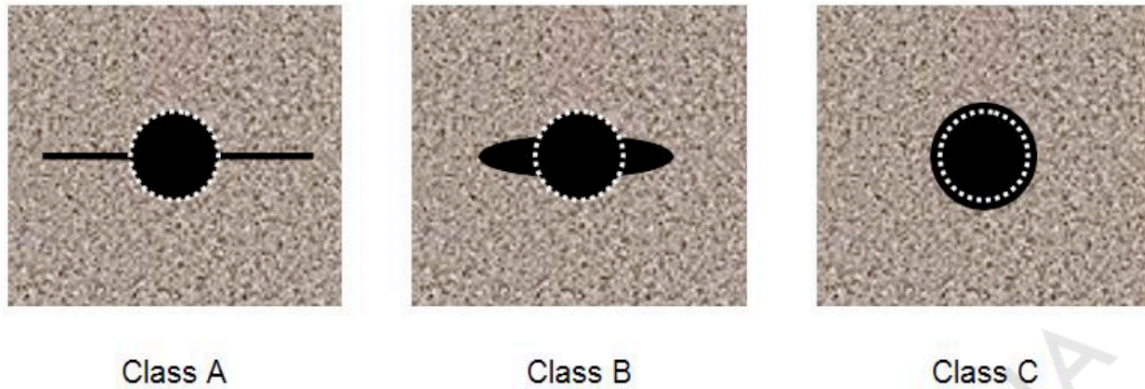


Figure 2.3 Schematic illustrations of the failure modes typical for the three rock classes. The white dotted lines indicate the initial size and shape of the cavity (Fjær et al, 2008)

Class A failure is characteristic of well-cemented, low-porosity sandstones that behave in a brittle manner. In this mode, the high tangential compressive stress concentrated at the borehole wall induces localized tensile fractures that propagate parallel to the borehole axis. The failure initiates small-scale spalling or concave openings but quickly coalesces into well-defined, slit-like breakouts. These features are often planar and can extend vertically along the full height of the specimen, representing a classic tensile splitting failure mechanism under a globally compressive stress field (Fjær et al., 2008). Class B failure represents the archetypal ductile shear failure mode. It occurs when the stress concentration exceeds the rock's shear strength, leading to the formation of conjugate shear bands that intersect at the borehole wall. This process results in the development of a localized breakout, often with a characteristic “*V-shaped*” or “*dog-eared*” cross-section. The material within this failed zone is typically crushed and disaggregated. Unlike the brittle mode, the vertical extent of these breakouts is limited, as the failure is confined to the region of maximum stress concentration (Fjær et al., 2008). Class C failure is a compactive mode unique to high-porosity, weakly consolidated sandstones. In this case, failure is not driven by shear but by volumetric compaction when the stress path reaches the material's compactive yield limit, or the “cap” on the modelled failure envelope. The primary mechanism is pore collapse and inter-granular crushing. Macroscopically, this manifests as a uniform and distributed closure of the

borehole rather than a distinct breakout. This mode is particularly relevant to sand production, as the disaggregation of the grain structure results in a zone of loose, cohesionless particles at the borehole wall that are readily mobilized by even minimal fluid flow (Fjær et al., 2008).

2.2.5 Sand erosion effects on production

Hydrocarbons are organic compounds that serve as the primary global source of energy, typically extracted as crude oil or natural gas. The formation of these compounds is a protracted geological process, initiated by the deposition and burial of organic matter. Over millions of years, increasing temperature and pressure transform this material into kerogen, which subsequently undergoes thermal cracking, or catagenesis, to generate fluid hydrocarbons. Following generation, these buoyant fluids migrate through carrier beds until they are intercepted by a trapping mechanism, accumulating within the pore space of a reservoir rock. Consequently, hydrocarbon reservoirs are typically found at significant depths, where the in-situ stress state and pore pressures are substantial. The engineering challenge of hydrocarbon recovery is therefore twofold: it involves not only accessing these deep resources but also managing the geomechanical response of the reservoir rock to the disturbances, such as drilling and fluid extraction, that are inherent to the production process. It is this geomechanical response, particularly in weakly consolidated formations, that gives rise to critical production challenges when producing sand particles (Fjær et al., 2008).

The operational and economic consequences of sand production are directly correlated with the volumetric rate and duration of the solid influx. A critical distinction is made between manageable and problematic sanding regimes. Manageable sanding typically involves transient or intermittent production of fine particles at low concentrations which does not threaten the continuity of production. On the other hand, problematic or continuous sand production is characterized by a sustained, high-volume influx of solids. This regime poses a severe threat to the entire production system. Downhole, large volumes of sand can accumulate, leading to a partial or complete filling of the wellbore that necessitates costly workover interventions. If transported to the surface, these solids cause aggressive erosional wear on chokes, valves, and pipelines ultimately leading to production shutdowns. Finally, of high importance is the logistical and environmental challenges of managing and disposing of large volumes of contaminated solids at the surface. The treatment and disposal of these materials require strict adherence to environmental regulations, often involving complex and costly processes. In some cases, the financial burden of

complying with these procedures can be economically unsustainable, with companies spending billions of dollars annually on preventive measures and waste management systems (Subbiah et al., 2021).

Due to these reasons, companies are continuously seeking solutions or investing funds, aiming to understand and mitigate the mechanisms causing or escalating sand production to prevent the aforementioned unwanted problems and avoid operational disruptions and catastrophic failures. Sand production is fundamentally a coupled hydro-mechanical process that requires the satisfaction of two conditions. The first is a prerequisite of mechanical failure, where the stress state around the wellbore exceeds the cohesive strength of the reservoir rock, leading to disaggregation of the grain structure. The second is hydrodynamic transport, where the drag forces exerted by the flowing reservoir fluids erode and carry away the failed rock fragments. The hydrodynamic forces alone are typically insufficient to damage intact or cemented rock but act primarily on material that has been pre-conditioned for failure by mechanical stresses (Li et al., 2018; Fetrati and Pak, 2020). The severity and dynamics of this erosional phase are intrinsically linked to the reservoir's production drive mechanism, which provides the energy for fluid flow. While mechanisms such as solution-gas drive or gas-cap expansion contribute to flow, the introduction of water into the near-wellbore region, either from a natural aquifer or through water injection, represents a significant escalation in sanding risk. Water's influence is twofold, as it detrimentally affects both stages of the sand production process. First, water can induce geochemical weakening of the rock matrix by dissolving mineral cementation or by promoting the swelling of certain clay minerals, thereby reducing the rock's intrinsic cohesive strength and making it more susceptible to mechanical failure. Second, due to its higher density and viscosity compared to hydrocarbons, water flow generates greater hydrodynamic drag forces, enhancing its capacity to erode and transport the already failed material. In this context, the influence of water can be categorized into three key mechanisms that significantly increase the propensity for sand production (Bellarby, 2009; Fjær et al., 2008):

1. Geochemical Weakening and Reduction of Apparent Cohesion: The intrinsic cohesive strength of many sandstones is highly sensitive to increasing water saturation. This geomechanical degradation stems from several mechanisms that operate at the grain scale. First, the presence of water can trigger the swelling of authigenic clays, such as smectite, which may be present as grain coatings or pore-filling cement. This swelling physically disrupts the inter-granular bonds and

reduces the overall strength of the rock fabric. Second, water can chemically dissolve soluble mineral cements, such as calcite, further weakening the rock frame. Finally, in partially saturated rock, negative capillary pressures (suction) at grain contacts provide a significant component of “apparent cohesion”. As water saturation increases, these capillary forces are neutralized, diminishing the rock's resistance to shear failure. This degradation is notably non-linear, with the most substantial reduction in strength often occurring during the initial increase in water saturation, typically within the first 20% (Aadnoy and Looyeh, 2011).

2. Loss of Capillary Cohesion in the Disaggregated Zone: Within the mechanically failed and disaggregated rock zone adjacent to the wellbore, a different capillary mechanism exists. Here, the sand grains are no longer a cemented mass but a granular pack. In a partially saturated state, small volumes of connate water form “pendular bridges” between the loose grains. The surface tension within these bridges creates an effective cohesive stress that can hold the failed material in place, analogous to the stability of damp sand. However, the arrival of a water-front during production, the water breakthrough, dramatically alters this condition. As the pore space becomes fully saturated with water, these pendular bridges are eliminated, and the negative capillary pressure dissipates. This loss of apparent cohesion effectively transforms the failed mass from a weakly stable solid into a cohesionless granular slurry, which is then readily mobilized and transported by even minimal fluid flow (Gholami et al., 2016).

3. Pore Pressure Transients and Increased Hydrodynamic Drag: The movement of a water-front through the reservoir introduces significant dynamic effects. Because water typically has a higher viscosity and density than the hydrocarbons it displaces, a steep, localized pore pressure gradient develops at the leading edge of the displacement front. This transient pressure pulse acts as a destabilizing body force on the already weakened or failed sand near the wellbore. Furthermore, the higher viscosity and density of water mean that for a given flow velocity, it exerts significantly greater hydrodynamic drag forces on individual sand grains compared to oil or gas. The combination of this transient pressure pulse and the amplified drag forces can trigger sudden, high-volume sand production events that are often observed to be coincident with water breakthrough at the production well (Dusseault & Santarelli, 1989; Papamichos, 2010).

2.3 Sand production experiments – The hollow cylinder tests

The hollow cylinder test (HCT) is an integral part of experimental geomechanics for investigating complex near-wellbore phenomena, including borehole stability, plastic zone development, and most critically for this research, the initiation and propagation of sand production (Rahmati et al., 2013). The primary advantage of the HCT is its ability to replicate the essential physics of the near-wellbore stress state in a controlled laboratory environment. By subjecting a cylindrical rock specimen with a central borehole to independent axial, radial, and internal fluid pressures, the test creates the steep stress gradients and radial flow convergence that are characteristic of a producing wellbore, providing an invaluable physical analogue for validating theoretical and numerical models (Cerasi et al., 2005; Rahmati et al., 2013).

The experimental apparatus typically consists of a high-stiffness triaxial loading frame equipped with a confining chamber capable of withstanding high pressures as shown in Figure (2.4). The rock specimen, a carefully prepared cylinder with a concentric inner borehole, is jacketed with an impermeable membrane and placed within the chamber.

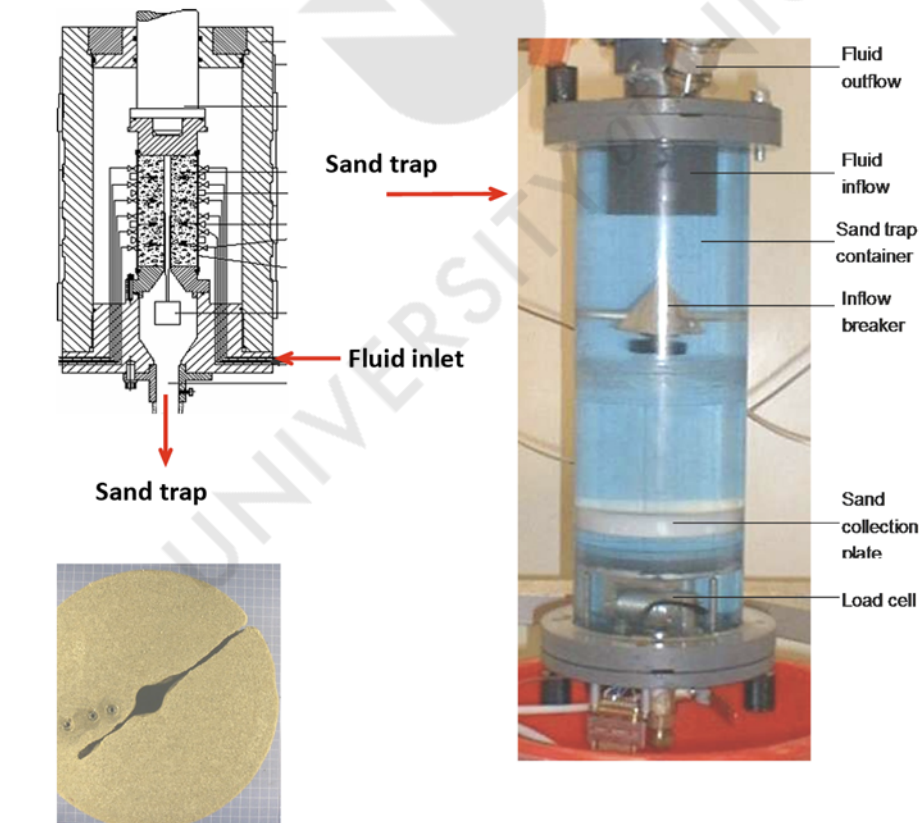


Figure 2.4 The Hollow Cylinder Test apparatus (Cerasi et al., 2005)

The setup allows for the independent application of an axial stress (σ_{axial}) and an external radial (confining) stress (σ_{radial}). These two stresses are used to simulate the vertical and horizontal far-field stresses experienced by the reservoir rock in-situ. A critical feature of the test is the ability to apply stress anisotropy (i.e., $\sigma_{\text{axial}} \neq \sigma_{\text{radial}}$), which is essential for inducing the shear failure mechanisms that precede sand production (Rahmati et al., 2013). In addition to the mechanical loading system, the apparatus incorporates a sophisticated fluid circulation system. This system allows for the injection of a fluid (usually a low-viscosity oil, brine or even water in some cases) into the specimen from its outer cylindrical boundary. The fluid permeates through the porous rock matrix and is produced from the inner borehole. By controlling the upstream and downstream pressures, a precise pore pressure gradient and a corresponding fluid flux are established, simulating the drawdown pressure that drives hydrocarbon flow into a well.

A typical sand production test protocol begins with the saturation of the specimen and its consolidation under isotropic or anisotropic stress conditions representative of the initial reservoir state. The main test phase then proceeds through a multi-stage loading path designed to probe the rock's failure envelope (Rahmati et al., 2013). The external stresses are typically increased in a step-wise manner. At each stress level, a fluid flow cycle is initiated, and the system is monitored for sand production as shown in Figure (2.5).

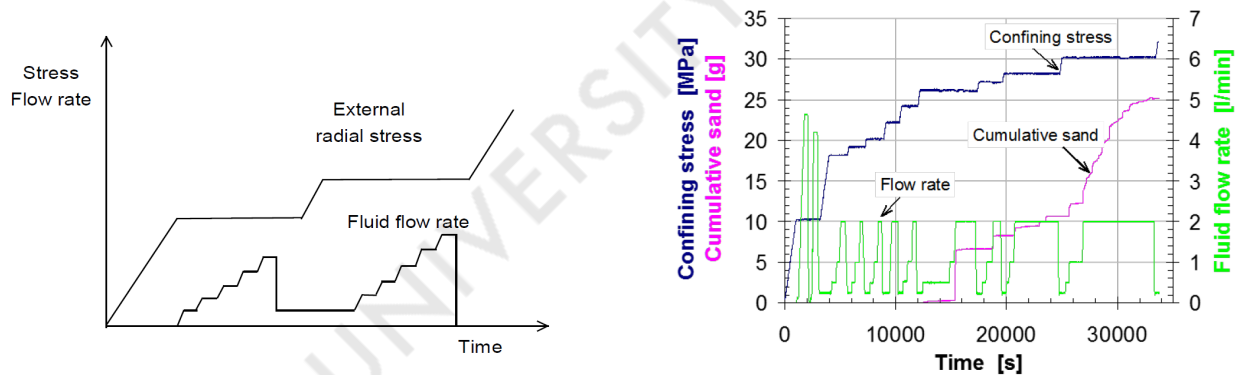


Figure 2.5 Typical flow boundary conditions and data collection in HCT (Cerasi et al., 2005)

The effluent fluid exiting the inner borehole is collected continuously or at discrete time intervals. The solid particles within this fluid are then separated and weighed to quantify the mass of produced sand. The flow is often maintained until sand production ceases or stabilizes, after which the confining stress is increased to the next stage and the process is repeated. This staged methodology allows for the determination of the critical stress and drawdown conditions required

to initiate and sustain sand production (Cerasi et al., 2005). Throughout the experiment, a comprehensive dataset is acquired. This includes the continuous monitoring of applied stresses, axial and radial deformations of the specimen, fluid flow rates, and the differential pressure across the sample. The primary output, however, is the cumulative mass of produced sand plotted against time. This sand production curve provides a direct, quantitative measure of the rock's erosional response to the applied hydro-mechanical loads. It serves as the definitive experimental benchmark against which the predictive capabilities of the analytical and numerical models, developed in this work, to be rigorously evaluated (Wang et al., 2005; Rahmati et al., 2013).

Finally, the test's ability to simulate complex multiaxial stress conditions allows for a deeper and more realistic evaluation of reservoir performance. This includes studying the initiation of sand production, formation compaction, and the evolution of erosion pathways under changing production regimes. These evaluations contribute to more innovative reservoir management, helping to extend the productive life of oil and gas wells (Rahmati et al., 2013).

2.4 The Evolution of Sand Production Modelling

The phenomenon of sand production from hydrocarbon production from weak rock formations represents a persistent and complex challenge for the oil and gas industry. As explained earlier, sand erosion is fundamentally a coupled hydro-mechanical process, that involves intricate interactions between fluid flow dynamics, rock deformation, material failure, and particle transport mechanisms (Vaziri et al., 2002a; Papamichos et al., 2001).

During the last few decades, research efforts have progressively deepened the understanding of this complicated challenge, moving from initial qualitative observations to highly sophisticated multi-physics computational models. This section intends to provide a thorough review of the development and advancements of sand production prediction modelling. The evolution of the research is categorized into distinct fields, highlighting experimental methodologies, numerical modelling techniques, and analytical frameworks to describe as best as possible the physics of sand erosion (Yi et al., 2005). Particular emphasis is placed given in distinguishing contributions in experimental, numerical and analytical showing the strengths, limitations, and interdependencies of each approach. This structured perspective aims to contextualize the current state of knowledge, identify persistent challenges, and emphasize the need for continued innovation in this critical area of petroleum geomechanics (Dusseault and Santhelie, 1989).

2.4.1. The foundations of erosion

The early stages of sand production research were characterized by efforts to establish a fundamental understanding of the conditions leading to sand onset, with the objective of providing operators with actionable decision-making frameworks. These early investigations were predominantly informed by field observations and guided by fundamental principles of rock mechanics and fluid flow, aiming to deliver preliminary operational guidelines (Geilikman and Dusseault, 1997). During this formative period, several defining characteristics shaped early predictive approaches. Operators typically relied on empirical operating envelopes, applying conservative production constraints, such as rate throttling or drawdown limits, based on experiential knowledge rather than engineering insight. Wellbore stability was evaluated using simplified models, frequently employing the Mohr-Coulomb failure criterion (describing shear failure of granular material) within a linear elastic framework (Labuz and Zang, 2012). These models generally neglected more realistic phenomena such as plastic deformation, stress anisotropy, and poroelastic coupling. The conceptual foundation for what would later be formalized as the Critical Drawdown Pressure (CDP) was also implicitly recognized, albeit without rigorous definition or calibration (Wang et al., 2005).

Diagnostic technologies at the time, lacked sophistication. Sand detection was limited to surface-based observations, including sand probes, separator sampling, and visual inspections, methods that provided delayed, intermittent feedback with no real-time or in-situ capability (Tronvoll et al., 2001).

Despite their limitations, these early developments established key terminology (e.g., onset versus rate, failure envelopes, drawdown control) and organizational workflows that subsequent research would formalize and quantify. However, such approaches lacked integrated hydro-mechanical coupling, did not enable sand rate prediction, and proved difficult to generalize across diverse reservoir conditions. These constraints ultimately laid the groundwork for the emergence of geomechanics-based and, later, fully coupled multi-physics frameworks at later stages (Wan and Wang, 2004).

2.4.1.1 Initial Experimental Contributions

During the early stages of research, experimental work was primarily qualitative or semi-quantitative, aimed at identifying the fundamental physical mechanisms responsible for sand initiation. Laboratory setups were typically simplified, prioritizing the demonstration of the phenomenon over the accurate quantification of sand volumes or production rates. One notable area of investigation was the concept of a "sand arch" where a stable grain configuration that could form around an opening and serve to prevent sand influx.

Hall and Harrisberger in (1970) conducted pioneering work in this area, qualitatively identifying the conditions necessary to form and sustain a stable arch over an opening, even under relatively high stresses. Their insights were crucial in conceptualizing the self-stabilizing mechanisms of granular media. (Hall Jr. & Harrisberger, 1970). Following their work, Bradly and Risnes (1981) further elaborated on the effect of flowing fluid on the stability and eventual failure of such sand arches. These early experiments, while not providing detailed quantitative data on sand production rates, helped establish the conceptual framework for subsequent research by identifying the critical role of both mechanical stability and hydrodynamic forces. The primary limitation of these early experimental efforts was their qualitative nature and the inability to capture the dynamic, time-dependent aspects of sand production or the actual volume of solids produced. (Bratli & Risnes, 1981)

2.4.1.2 Initial Numerical and Analytical Models

In parallel to the experimental efforts, analytical and numerical models emerged, predominantly based on simplified mechanical assumptions. These models were computationally less demanding and focused almost exclusively on predicting the onset of sand production, often neglecting the subsequent volumetric production. Many of these models employed elastic brittle failure criteria. Such works were implemented to predict sand erosion onset by considering the failure of the sand matrix. While providing quick estimates, these models often led to excessive stress concentrations at the borehole wall due to their assumption of brittle failure, resulting in overestimation of the initial sand production conditions. They were typically used as preliminary screening tools in relation to production parameters and in-situ stresses, but their predictions were limited in accuracy unless combined with apparent strength models. The inherent simplicity of these elastic models

meant they could not capture the complex, non-linear behavior of reservoir rocks under stress and fluid flow (Nordgren et al., 1977; Coates & Denoo, 1981 and Risnes et al., 1982).

Geertsma (1985) made significant analytical contributions by studying the geomechanical aspects of well completions and sand production during both injection and production cycles. His work provided a more rigorous analytical framework for understanding stress distributions around the wellbore. Empirical approaches also gained attraction, attempting to establish correlations between easily measurable field parameters and sand production. Stein and Hitchie (1972) for instance, developed a method for predicting sand production rates based on the evaluation of formation strength from log data and offset well conditions. While practical for quick field assessments, a major defect of these empirical methods was their inherent field-dependency. Results obtained from a single reservoir often could not be reliably extrapolated to another, limiting their general applicability and predictive power. (Stein & Hilchie, 1972)

The contributions of these research works laid the conceptual groundwork but was constrained by computational limitations and an early-stage understanding of the complex coupled processes. Models were largely uncoupled, simplified, and focused on onset prediction, leaving the challenge of volumetric and rate prediction largely unaddressed.

2.4.2. Development of Coupled Processes and Advanced Experimentation

The initial studies on sand erosion marked a significant shift in sand production research, driven by the recognition that a comprehensive understanding is required, accounting for the strong coupling between fluid flow and mechanical deformation. Following the initial studies, provided the ground for the development of more controlled and quantitative experimental techniques, alongside the first attempts that aimed at integrating hydro-mechanical coupling into numerical models.

2.4.2.1 Advanced Experimental Studies

Experimental works have advanced, shifting from qualitative observations to quantitative measurements of sand production and detailed analyses of cavity development. The hollow cylinder test became a common laboratory setup, enabling controlled application of stress and fluid flow conditions to more accurately simulate the near-wellbore environment.

Key studies focused on the dynamics of cavity evolution and the role of plastic yielding. Alsiny et al., (1992) conducted visual observations to examine cavity shape and the development of localized deformation modes, offering insights into the evolving geometry of the damaged zone. Kooijman et al., (1992) carried out large-scale experimental tests with multiple perforations using natural outcrop rock, allowing the study of the combined effects of effective stress, drawdown, and water cut on sand production. Cook et al., (1994) contributed by monitoring sand production while systematically varying either stress at a constant flow rate or flow rate at a constant stress, providing data on the sensitivity of sand production to operational parameters. The concept of cavity failure and its role in sand production was further explored by Van den Hoek et al., (1996), who examined the influence of different cavity sizes.

The influence of various rock and fluid properties on sand production has gained increased attention. Behrmann et al., (1997) assessed sand production in relation to confining stresses, flow rate, and fluid type, emphasizing on the complex, multi-factorial nature of the issue. Tronvoll et al., (1997) carried out experimental investigations specifically on the effect of oil production in ultra-weak sandstones, tackling challenges in highly unconsolidated formations. Unander et al., (1997) concentrated on the effects of flow geometry, illustrating how the configuration of fluid pathways impacts particle mobilization while Al-Awad et al., (1999) designed experiments to explore the effects of confining pressure, flow rate, and fluid viscosity on sand production in unconsolidated sandstone, supplying vital data for understanding fluid-rock interactions. Ewy et al., (1999) simulated wellbore failure in laboratory conditions by applying in-situ stresses to a perforation, establishing a direct link between mechanical loading and failure. Van den Hoek et al., (2000) extended these studies by examining the long-term stability of horizontal wellbore completions and how rock failure in the near-wellbore region affects well productivity, which is a key consideration for the design of horizontal wells.

A key experimental finding, influential for subsequent numerical modelling, was presented by Papamichos et al., (2001). Through laboratory experiments on hollow cylinder tests, they claimed that plastic yielding, resulting from external loads, leads to micro-cracks that serve as erosion starter points. This proposition, while later challenged, was instrumental in guiding the development of coupled models that considered material degradation. Walton (2001) conducted experimental investigations on perforating unconsolidated sands, measuring produced sand during both perforating and subsequent fluid flow. Chin and Ramos (2002) investigated the effect of rock

strength, flow rate, oil viscosity, and producing time on sand production, emphasizing on the dynamic nature of the process. The detailed comparison of cavity shape and failure mechanisms around vertical perforations in high-porosity sandstones was performed by Haimson & Lee, (2004). At a later stage, Wu et al., (2005). investigated experimental perforation collapse tests to explain and quantify the water-cut effect on cavity failure, a critical factor in mature fields. Nouri, et al., (2006) focused on obtaining real-time sand production data, which is essential for validating dynamic models.

Despite these significant advancements, a major drawback of these experimental methods remained the inherent simplifications and assumptions in laboratory setups. These approximations, especially regarding rock type selection (e.g., synthetic versus natural samples), perforation size, fluid characteristics, confining stress and fluid pressure application, could result in nonrealistic outcomes, particularly when projecting to complex field-scale studies. The limited capacity to accurately simulate multi-phase flow conditions also posed a challenge.

2.4.2.2 The Continuum Approach with Numerical and Analytical Modelling

This section discusses the concerted efforts to develop fully coupled numerical models, moving beyond simple elastic assumptions to incorporate elastoplastic material behavior and the complex interaction between fluid flow and rock deformation. The goal was to accurately represent the progressive damage to the rock, which was often overlooked by earlier models. The foundational theory for hydrodynamic erosion of sandstone, based on filtration theory, was proposed by the seminal work of Vardoulakis et al., (1996). This work, introduced the concept of the "*sand production coefficient λ* " (a phenomenological coefficient controlling erosion rate), was a critical step in integrating the hydro-dynamical aspect into the mechanical framework. While it initially did not solve the equilibrium equation for the solid phase, it provided a basis for quantifying erosion. At this point is important to note that the investigation of this sand production coefficient is one of the objectives of this research work. Following this seminal work, Stavropoulou et al., (1998) and at a later stage Papamichos et al., (2001) advanced the Vardoulakis et al., (1996) purely hydromechanical model by integrating the poromechanical behavior of the solid-fluid system with the erosion behavior of solids caused by fluid flow. This represented a significant step towards fully coupled models. These studies, along with contributions from Vaziri et al., (2002a), started

to include the complete strength hardening and softening behavior of sandstone in their models, aiming to reflect the progressive damage observed in reality.

The first fully coupled numerical models often focused on calculating the onset of sand production or the initiation of mechanical failure. These models while incorporating some coupling, sometimes simplified by ignoring critical flow rates or not simulating material softening (Skjaerstein et al., 1997; Papamichos & Malmanger, 1999; Yi, 2001). The application of bifurcation theory also gained popularity. Papanastasiou and Vardoulaksi (1992) were among the first to use bifurcation theory to examine cavity stability and its connection to sand production, employing finite elements for stress and strain analysis to study strain localization within the post-bifurcation region. Later, Tronvoll et al., (1993) modelled asymmetrical problems using FEM to identify factors influencing non-axisymmetric bifurcation modes. Papanastasiou and Zervos (1998) performed 3D numerical simulations to investigate how perforation orientation affects stability and failure, emphasizing that perforating parallel to the minimum horizontal stress could increase the risk of failure.

The Mohr-Coulomb (MC) model became a common yield function in sand models because of its ability to model cohesive frictional material and providing always the maximum output of plastic yielding which is very convenient for investigating worse case scenarios. Although some researchers simplified their models by omitting compression yielding (cap failure) attempted to use elastoplastic models for reaching realistic results yet faced difficulties in accurately modelling strain localization phenomena, which are inherently discontinuous. This fundamental inefficiency resulted in the inability to reflect size dependency in results based on numerical mesh design, causing predictions to vary considerably with mesh refinement (a.k.a. mesh dependent problems).

Some studies conducted numerical analysis specifically for sanding in injector wells, recognizing that sanding mechanisms in these wells could differ significantly, potentially involving phenomena like water hammer waves, and often leading to high sand production within short periods (Vaziri et al., 1998; Yi et al., 2001). Despite these advancements, the computational cost of these fully coupled models remained a significant limitation, restricting their use in large-scale field problems to date. Moreover, their dependence on continuum assumptions meant they found it difficult to accurately represent the discrete nature of particle detachment and transport.

2.4.3. Advancements in Multi-scale and Comprehensive Approaches

The research performed up to the early 2000s marked a new era in sand production research, characterized by the widespread adoption of advanced numerical techniques, including both refined continuous and emerging discontinuous methods devoting increased focus on integrating diverse data sources and developing more comprehensive predictive frameworks. The main goal shifted from simply predicting sand onset to providing more precise and dependable quantitative forecasts of sand volume and rate.

2.4.3.1 Refined Continuum Based Numerical Models)

Numerical models based on continuum mechanics have become increasingly sophisticated, incorporating more realistic constitutive models, advanced coupling schemes, and more refined sanding criteria. Researchers advanced towards more sophisticated elastoplastic models capable of capturing the complete spectrum of rock behavior, including strength, hardening and softening. Papamichos & Vardoulakis (2005) developed a model based on nonlinear elastoplasticity, accounting for nonlinear stress-dependent elasticity, friction hardening, and cohesion softening, fully coupled with single-phase flow. Their model utilized a porosity diffusion-type law, resulting in a sand rate that decreased over time as the erosion zone enlarged. Wang et al., (2005) performed a fully coupled single-phase flow analysis based on erosion mechanics using FEM, applying it to 2D plain strain geometry for both open-hole and perforated casing completions. The evolution of sanding criteria became also important because it was derived from experimental findings calibrated to meet realistic conditions and extrapolate for field-scale applications. Research works, based on laboratory experiments, observed the slit cavity evolution pattern during sand production, informing new criteria (Haimson & Lee, 2004; Papamichos, 2006). Detournay et al., (2006) proposed a model to predict the formation of slit channels based on a critical flow rate, modifying Vardoulakis et al., (1996) erosion law with the addition of a critical flow rate as a function of grain size. They assumed continuous sand production until reaching a critical porosity, beyond which the material suddenly collapsed. This model, applied to 2D plain strain geometry, could predict various erosion features such as surface spalling and small burst events.

The inclusion of more comprehensive sanding criteria, such as tensile and force balance on elements, gained prominence. Many research works developed models where elements satisfying

the sanding criteria were removed from the mesh, assuming the cavity could grow adjacent to the wellbore (Vaziri et al., 2002a; Vaziri et al., 2008, Nouri et al., 2006 and Nouri et al., 2007). A notable advancement came from Kim et al., (2011), who calculated sanding conditions and used a sanding criterion based on the force balance on each element. This approach achieved a good match with experimental data and had the significant advantage of not requiring a specific calibration parameter for sanding. In their criterion, hydrodynamic forces generated by the pore pressure gradient between element faces drove sand production, while resistance forces from vertical and tangential stresses and the friction coefficient retained the elements in place.

A significant critique on coupled hydromechanical erosion models is their conflicting assumptions of rigid porous media for mass balance versus deformable for equilibrium. To address this, efforts were made to consistently divide porosity changes into parts related to volume variations caused by erosion and deformation due to stress changes. Li et al., (2018) presented a finite element-based approach for simulating the realistic sanding process, explicitly considering coupling between mechanical failure and hydrodynamic erosion of the rock. This model aimed to predict local stress and strain distribution, fluid velocity fields, and the total sand production amount during a well's production history. However, they acknowledged that calibration with experimental results and field data was still needed to provide further confidence for field applications. While axisymmetric models, such as those used by Azadbakht et al., (2012) based on simplified simulations assuming ring-shaped perforations, they inherently influenced the pressure gradient and mechanical response around the perforations. Crucially, these models failed to recognize the significant influence of perforation direction, which Papanastasiou and Zervos (1998) had already demonstrated through 3D numerical simulations could be vital for perforation stability and failure.

2.4.3.2 Multi-methods and Empirical Approaches

Multi-methods integrate various approaches by combining the strengths of various methodologies to produce more robust and reliable predictions. Empirical methods also continue to develop, incorporating more advanced data analysis. Qiu et al., (2006) carried out sand prediction analyses using a method that combines easy-to-measure lab data, log data, analytical calculations, and empirical techniques, all supported by results from a previously run rigorous numerical code. This multi-faceted approach aims to utilize the advantages of each method while reducing individual

limitations. Osisanya, (2010) provided a comprehensive overview of various techniques for predicting sand production, including the use of production data, well logs, laboratory testing, acoustic and intrusive sand monitoring devices, and analogy, describing detailed methodologies and data requirements supported by examples and case studies from around the world.

An empirical work by Khamsehchi & Reisi, (2015) used the relationship between shear modulus and bulk compressibility to predict sand production based on field observations and well data. Abbas, et al., (2019) carried out sand production prediction analyses by developing a method that combines log data, measurable experimental data, analytical relationships, and empirical techniques. Although these modern empirical methods are more advanced, their results still tend to depend on field conditions, which limits their direct applicability. Shabdirova et al., (2019) proposed a prediction model for weak sandstone reservoirs, estimating the volume of produced sand as the volume of the plastic zone of the failed materials surrounding the perforation tunnels. This model, while simple, showed good agreement with measured sand volumes in a local oil field, though it did not consider material strength degradation and assumed homogeneous and isotropic sandstone. On the other hand, Araujo-Guerrero et al., (2022) presented a mathematical model to define the conditions of sanding onset and to predict and quantify the sand rate. Their work emphasized the indispensability of elastoplastic constitutive models and demonstrated a clear relationship between sanding and plastic strains. They also highlighted that elastic constitutive relationships are strongly limited when modelling sanding-prone formations.

2.4.3.3 Real-time Monitoring and Advanced Experimental Techniques

Alongside advancements in numerical modelling, experimental techniques also grew more sophisticated, integrating real-time monitoring and advanced imaging technologies to capture the dynamic processes of sand production with greater accuracy. This includes studies by Wu et al., (2016), who used real-time monitoring with X-ray CT scanning to observe the development of cavities and sand production. Xing et al., (2024) carried out experiments under various compression scenarios for validation, using Digital Image Correlation (DIC) and 3D X-Ray Computed Tomography (CT) for detailed analysis of deformation and microstructural changes. Zivar et al., (2019) examined sand production stages and permeability changes during the process, offering insights into the evolving reservoir properties. Deng et al., (2019) investigated the critical

production pressure difference and the critical flow rate of sand production in unconsolidated sandstone reservoirs in deep-water gas fields, addressing specific operational issues.

Further experimental studies were used to refine the understanding of specific factors. Choi (2011) investigated perforating unconsolidated sandstone and measured the amount of produced sand during perforating and post-shot flow, as well as the mechanisms of sand production and the mechanical characterization of caverns. Fattahpour et al., (2012) introduced an innovative method to continuously measure produced sand and studied sand production in terms of rock strength and perforation size. Rahmati et al., (2013) conducted tests to simulate sand production from a perforation cavity while Wu & Choi (2012) studied the fundamental mechanisms of sand production and perforation failure development, investigating the failure condition of a hollow cylinder using various geomechanical tests. Mier et al., (2013) examined the influence of borehole diameter on the formation of borehole breakouts, while Ranjith et al., (2013) investigated factors influencing sand production from a screen after completion. Despite the sophistication, and due to and many approximations in sand production analysis, results between field observations and experiments do not always agree.

2.4.4. Filtration Theory as an Erosion Law and Experimental observations

Gravanis et al. (2015) presented a comprehensive framework for modeling sand production by coupling mechanical failure of the rock matrix with hydraulic erosion processes at the pore and grain scale. The study emphasizes that sand production is not governed solely by stress-induced failure, but also by fluid-driven detachment and transport of grains once mechanical instability is initiated. By integrating elastoplastic deformation, damage evolution, and erosion laws linked to fluid flow, the authors demonstrate how progressive material degradation can lead to sustained sand production under reservoir conditions. Their hydro-mechanical approach provides important insights into the interplay between stress redistribution, permeability evolution, and fluid forces, offering a more physically consistent basis for predicting sanding onset and post-failure behavior in weakly consolidated formations (Gravanis et al., 2015).

Shahsavari et al., (2021b) referred to sanding criteria, failure reasons, and numerical techniques that embody the concept of erosion. Their work emphasized on how various models and experimental setups recognize and simulate the detachment and transport of sand particles. Specifically, they explored the specific forces driving this process, often mentioning seepage

failure and drag force as direct physical causes of material dislodgement caused by fluid flow. These terms highlight the microscopic interaction between the flowing fluid and the solid rock matrix.

In numerical models, several simulations, especially those using finite element (FE) or finite difference (FD) methods, explicitly specify erosion as the criterion that triggers sand production. For example, Papamichos et al., (2001) employed a poro-elastoplastic FE model that identifies erosion as a key factor in determining when sand will be produced, highlighting the weakening of the rock's structural integrity due to fluid action. In turn Nouri et al., (2004) and Rahmati et al., (2013) used 2D FD models with erosion laws as criterion, indicating that their computational frameworks are designed to consider material loss driven by fluid flow. These models frequently associate erosion with specific mechanical thresholds, such as plastic strain, where exceeding these thresholds results in particle detachment.

The explicit modelling of erosion is usually achieved through algorithms that remove or alter elements representing the rock matrix once specific sanding criteria are met, thereby computationally simulating material loss. For example, sanding begins from cavity face elements when the cumulative shear plastic strain reaches a critical value, which at that point the element is removed from the model. This removal, based on accumulated shear plastic strain, serves as a key numerical representation of the erosion process, indicating that once the rock experiences sufficient plastic deformation and weakening, it is regarded as dislodged and transported by the fluid.

Shahsavari et al., (2021b) emphasized the importance of deciding whether to remove or retain elements that meet the sanding criteria for accurately predicting wellbore geometry evolution and sand production volume. Two main methods enable this material removal. Firstly, direct element deletion when criteria are met (a simple approach where elements indicating failure are removed from the mesh), and secondly, more advanced adaptive meshing techniques that dynamically modify the computational mesh while deleting elements from the cavity surface, allowing for a better representation of the evolving wellbore and a more precise simulation of the erosion front.

Experimental observations also support erosion as a primary failure mechanism, especially in tests designed to mimic wellbore conditions. Experiments involving hollow cylinders subjected to fluid flow, demonstrate erosion leads to wellbore breakouts or the collapse of sand arches. This

indicates that sustained fluid flow physically removes material from the wellbore face, resulting in instability (Meier et al., 2013; Ranjith et al., 2013). Other studies by Wu & Choi (2012) and Unander et al., (1997) similarly attributed wellbore break-outs to erosion, sometimes in conjunction with mechanical failure. This indicates a combined effect where the rock's structural integrity is compromised, and fluid flow then dislodges the weakened material.

2.4.5. Current Challenges and Future Directions

Despite the significant advances in experimental, numerical, and analytical approaches, predicting sand production remains a formidable challenge. Several fundamental questions and weaknesses remain in existing models that pose inherent challenges in the problem of erosion, requiring ongoing research and innovation.

2.4.4.1 Numerical Models Weaknesses

Numerical models, while powerful in predicting capabilities, still grapple with several limitations:

- Many researchers tend to avoid incorporating critical-state-based constitutive models, possibly due to the difficulty in calibrating numerous parameters such as hardening and softening. This restricts the models' capacity to accurately represent the full range of material behavior under different stress paths (Wan and Wang, 2004).
- The selection of a sanding criterion (e.g., erosion, tensile, shear failure) often depends on the type of rock. Erosion criteria are more suitable for weak rocks, while stronger rocks require tensile or shear failure criteria. A universal criterion that applies across a broad range of rock strengths has yet to be developed (Papamichos et al., 2001 and Yi et al., 2005).
- The specific type of sand production (such as gradual erosion or sudden collapse) greatly affects the evolving cavity shape and the permeability distribution in the damaged zone. Current models often oversimplify these aspects, resulting in inaccuracies (Geilikman and Dusseault, 1997).
- Accurately simulating perforated wellbores requires a very fine mesh around the perforations, which makes 3D analysis computationally demanding and time-consuming. 2D models of perforations, using plane strain and axisymmetric assumptions, do not fully capture the precise mechanical and fluid flow responses, leading to significant simplifications or require higher order algorithms for upscaling (Wang et al., 2005).

- The complex behavior of sand arching, where particles form stable bridges that can temporarily prevent sand ingress, is difficult to accurately model in numerical simulations, especially within continuum frameworks (Vaziri et al., 2002a).

2.4.4.2 General Sand Production Prediction Weaknesses

Apart from particular model limitations, the inherent complexity of sand production poses general challenges:

- The complex nature of sand production that arises from the presence of numerous interdependent parameters (rock properties, fluid characteristics, stress state, wellbore geometry, production history) that are difficult to study thoroughly in experimental setups and under varying wellbore conditions (Papamichos et al., 2001; Yi et al., 2005).
- Each approach, either analytical, empirical, experimental or numerical has inherent assumptions and compromises concerning accuracy, complexity, size, number of elements, runtime, and the need for input data. These choices inevitably influence the reliability and applicability of the results (Dusseault and Santhelier, 1989; Vaziri et al., 2002b).
- Most numerical and analytical methods still rely on two-dimensional models, which include many simplifications and assumptions that may not hold true in a 3D field environment. While 3D models are more realistic, they introduce several shortcomings, such as significantly longer runtime, the common limitation of considering only one-cylinder perforation and often neglecting interactions among multiple perforations due to computational constraint (Rahmati et al., 2013; Wang et al., 2005).
- Actual field data often contain inherent uncertainties and measurement errors related to properties such as permeability, porosity, viscosity, rock strength, Poisson's ratio, and in-situ stresses ;). Overlooking these uncertainties in numerical simulations can significantly impact model predictions and bias subsequent engineering decisions (Acock et al., 2004; Veeken et al., 1991).
- There is a notable absence of a universally accepted benchmark for sand production criteria. This is mainly due to the diverse failure patterns, varying cavity shapes, complex hydrodynamic effects, erosion mechanisms, different stress states, and a wide range of rock types encountered in the field (Wan and Wang, 2004; Yi et al., 2005).

Despite these ongoing errors and uncertainties in laboratory and modelling studies, the simulation of actual sand production in certain field cases, such as a North Sea Reservoir with identical-sized elements, has demonstrated relatively acceptable accuracy. This level of predictive ability, even if sometimes within the range of 0.1–10 times the actual rate (can still be highly valuable for subsequent decision-making and sand control management in the field (Fuh & Nozaki, 2014; Kim & Sharma, 2012; van den Hoek & Geilikman, 2005; Wu et al., 2016).

2.5 Special Issues and Future Horizons in Sand Production Research

The future of sand production research is shifting from static, isolated diagnostic methods to dynamic, integrated multi-physics systems that focus on real-time prediction accuracy. Currently, research highlights the importance of coupled hydro-mechanical processes, particularly plastic yielding and material degradation. Moving forward, the field's development depends on refining these relationships to better represent the changing conditions of reservoir environments.

A primary focus of future inquiry will be the shift from considering erosion parameters, such as the sand production coefficient (λ), as fixed constants to viewing them as variables dependent on stress. This research has already demonstrated a strong power-law relationship between the erosion coefficient and applied far-field stress, indicating that future constitutive laws will shift toward non-linear, kinematic formulations capable of adapting to changing in-situ conditions. Incorporating stress-sensitive parameters into semi-analytical models enables researchers to produce computationally efficient solutions that connect numerical simulations with field-scale operational requirements.

Furthermore, the progression of numerical modeling is expected to lead to a broader adoption of Arbitrary Lagrangian-Eulerian (ALE) formulations. This methodology facilitates the simulation of large-scale surface erosion and the development of plastic zones without the numerical instabilities and mesh dependencies that have traditionally limited continuum models. As computational resources improve, advanced finite element models will increasingly be combined with real-time monitoring data to create responsive models of the near-wellbore environment.

Ultimately, the research trajectory is moving towards a comprehensive methodology wherein laboratory-derived insights—such as the impact of grain angularity, cementation quality, and thermal stresses—are seamlessly incorporated into large-scale reservoir management tools. By advancing from the binary prediction of "sanding onset" to the precise quantification of

volumetric sand rates throughout the entire lifespan of a well, the industry will be better equipped to design proactive sand control strategies and optimize well completions within increasingly complex geomechanical environments. This transition from observation to high-resolution prediction constitutes the foundation of future innovations in hydrocarbon recovery.

2.6 Conclusions

In this chapter we have attempted to present the major challenges in sand erosion that constitutes a major challenge in the hydrocarbon recovery industry, affecting operational efficiency, equipment integrity, and ultimately, economic sustainability. The chapter attempted a comprehensive overview of sand erosion, examining its fundamental mechanisms and the multifaceted factors influencing its occurrence. In this endeavor, we have investigated how the interaction of in-situ stresses and the inherent characteristics of reservoir rocks, especially for weak sandstones, influences a formation's tendency to produce sand.

The discussion emphasized the vital role of weak unconsolidated formations, where the breaking of grain-to-grain contacts under production-induced stress changes is the main cause of sand detachment. Additionally, in this chapter we have evaluated the impact of sedimentary rock formation and diverse fracture modes on sand erosion. The adverse effects of sand on production operations were elaborated. And have presented the areas of ongoing efforts to understand and mitigate this complex phenomenon, either with theoretical models or advanced experimental studies utilizing sophisticated techniques to simulate reservoir conditions and observe sand detachment mechanisms. Despite decades of research, effectively predicting and managing sand erosion still remains a vital area, requiring continuous innovation in monitoring, prevention, and remediation strategies to ensure sustainable and efficient hydrocarbon recovery.

CHAPTER 3-FEM MODELING OF SAND PRODUCTION

3.1 Preamble

This chapter aims to shed some light on the physics of the sand production coefficient λ with inverse length units (1/m) that arise in hydrodynamic models. It examines how this sand production coefficient λ depends on external stress conditions and how the combination of the confining stresses and the production coefficient affects the plastic zone (Λ) development. The analysis is conducted by means of hollow cylinder simulations with finite elements. The outcome of this analysis is a proposed expression dictating the strength of erosion on the confining stress around the hollow cylinder test, highlighting the significance of the sand production coefficient λ in modelling and simulation claiming that mathematically it expresses the strength of erosion.

To achieve this, the Arbitrary Lagrangian-Eulerian (ALE) methodology was utilized that is very convenient in finite element simulations, which accounts for the poro-mechanical coupling of the fluid-solid system for simulating hollow cylinder tests successfully. For the dependence of λ on external stress, a three-parameter model (a scale parameter, an exponent, and a stress parameter) have been proposed, which predict the erosion onset successfully by suppressing the transient regime successfully. This assumption on one hand is identified as a limitation in the simulations, but on the other hand it is quite accurate for hollow cylinder experimental tests. In other words, the transient regime (i.e., the time that the physical problem takes to stabilize sand production in the experiments is negligible).

Furthermore, the research demonstrated that the stress parameter is linked to the minimum stress required for plastic yielding to take place. As a result, the power law model that describes how λ depends on the depth of the plastic zone is also proposed and discussed.

3.2 Theoretical model of sand production

As mentioned in chapter 2, sand production is divided into two physical processes. On one hand, there is the poro-mechanical response of the rock sample, which is described by the theory of poroelastoplasticity, and on the other hand, the erosional response of the rock fabric, defined by filtration theories. Understanding these two coupled processes clarifies their synergy in sand production. The poro-mechanical response triggers grain failure and detachment by redistributing

stress within the porous structure. Subsequently, erosion mobilizes and transports the detached particles through the forces that are created by the permeating flowing fluid. These processes constitute a sequential mechanism: mechanical failure initiates erosion that in turn influences the stress state and permeability and porosity of the rock. This investigation assumes that the rock formation is composed by a uniform and isotropic rock with a single-phase fluid-saturated poromechanical system, considering fully dynamic conditions for rock deformation and fluid flow (Vardoulakis et al., 1996, Papamichos et al., 2001 and Gravanis et al., 2015).

The governing equations for **poro-mechanical response**, under quasi-static conditions, are as stated below (Papamichos et al., 2001, Papamichos E., 2020 and Shahsavari et al., 2021b):

$$\text{Equilibrium equations: } \sigma_{ij,j} = 0 \quad (3.1)$$

$$\text{Porous solid response: } d\sigma_{ij} = C_{ijkl}^{ep} d\epsilon_{kl} + \alpha dp \delta_{ij} \quad (3.2)$$

$$\text{Fluid continuity: } q_{i,i} = 0 \quad (3.3)$$

$$\text{Darcy's law: } q_i = -\frac{k}{\mu} p_{,i} \quad (3.4)$$

where σ_{ij} is the total stress tensor, ϵ_{kl} are the shear strain tensor, p is the pressure in the fluid, q_i is the fluid flux, C_{ijkl}^{ep} is the tangent poroelastoplastic stiffness matrix (a high order tensor), α is the Biot coefficient, k is the permeability, μ is the permeating fluid viscosity and δ_{ij} is the Kronecker delta.

For the **erosional response**, the governing equations are (Vardoulakis et al., 1996, Papamichos, E., & Stavropoulou, M. 1998 and Papamichos et al., 2001):

$$\text{Solids continuity: } \frac{\dot{m}}{\rho_s} = \frac{\partial \phi}{\partial t} \quad (3.5)$$

Equation (3.5) serves as a mass balance equation.

$$\text{Solids mass creation: } \frac{\dot{m}}{\rho_s} = \lambda(1-\phi)q_i \quad (3.6)$$

Equation (3.6) is the constitutive equation for sand production, mainly intended for oil wells.

$$\text{Kozeny-Carman: } k = k_0 \frac{\phi^3}{(1-\phi)^2} \quad (3.7)$$

Equation (3.7) depicts the significant variation of permeability with porosity and is the well-known Kozeny-Carman equation. Where $\dot{m}$ expresses the erosion rate (solids mass rate per unit of volume), ϕ is the porosity of the rock, ρ_s is the rock density, k_0 is the Kozeny-Carman parameter that is related to the initial permeability and k is the permeability.

Simulations using Equation (3.6) suggest that sand erosion, in the case of gas production, will generate significant amounts of sand due to low viscosity and consequently high flow rates. The physical content of a model based on the above is that the erosion rate is proportional to the fluid flux, and there is also a dependence on the remaining mass, which has been inferred by the factor $(1-\phi)$. Based on the filtration theories, as presented in the work of Vardoulakis et al., (1996) these are some reasonable assumptions for a constitutive erosion equation. The sand production coefficient λ has dimensions of inverse length (1/m). It is the proportionality constant of the constitutive law (constitutive parameter) and could be determined experimentally by sand erosion tests.

A significant number of researchers mention that λ is considered as an empirical constant/function which requires special treatment (Stavropoulou et al., 1998; Papamichos et al., 2001; Gravanis et al., 2015). For any given rock type, the experimental work of Papamichos et al., (2001) has shown that erosion onsets when the stress magnitude exceeds a certain threshold (i.e rock yielding), depending always on the fluid flow. When this threshold is reached, the material shows a softening behavior (Gravanis et al., 2015; Sarris et al., 2021). Based on the assumption that λ is a function of the plastic strain, the softening behavior is encoded in the constitutive equation (3-6) and presented by a number of researcher as well (Li et al., 2018; Papamichos et al., 2001; Fetrati & Pak, 2020; Sarris et al., 2021). Furthermore, by connecting erosion with plasticity, the essential weakness of the erosion starter point in the original formulation of Vardoulakis et al., (1996) is solved. This is explained that significant erosion from the bulk of the material will start

from the internal surface of the hollow cylinder as observed experimentally and mathematically it is the strain localization on the internal surface. The Vardoulakis et al., (1996) model assumed that erosion may start from anywhere inside the cylinder which is recognized as a limitation. Nevertheless, Vardoulakis et al., (1996) provided the very first seminal hydromechanical equations.

This formulation establishes a direct connection between the mechanisms of grain detachment and the constitutive description of erosion, thereby enabling the phenomenon to be represented within a coherent theoretical framework. By relating the sand production coefficient to measurable deformation parameters, such as shear or plastic strain, the model can be calibrated against laboratory and field data. This approach enhances the reliability of predictions across various rock types and fluid flow conditions, while also facilitating the systematic incorporation of material heterogeneity and anisotropy into the analysis of sand production.

The sand production coefficient as a function of the shear strains was originally proposed by Papamichos, E., & Stavropoulou, M. (1998) and later utilized for various applications by (Li et al., 2018; Papamichos et al., 2001; Fetrati & Pak, 2020; Sarris et al., 2021)

$$\lambda(\varepsilon^{pl}) = \begin{cases} 0 & \text{if } \varepsilon^{pl} \leq \varepsilon_{thr}^{pl} \\ \lambda_1 & \text{if } \varepsilon_{thr}^{pl} < \varepsilon^{pl} < (\varepsilon_{peak}^{pl} + \lambda_2/\lambda_1) \\ \lambda_2 & \text{if } \varepsilon^{pl} \geq (\varepsilon_{peak}^{pl} + \lambda_2/\lambda_1) \end{cases} \quad (3.8)$$

where ε^{pl} is the plastic shear strain, ε_{thr}^{pl} is the plastic shear strain value at which erosion is turned on, ε_{peak}^{pl} is the plastic shear strain at the maximum sand production coefficient, λ_1, λ_2 are calibration constants to be determined through calibrations as stated in the literature by (Li et al., 2018; Papamichos et al., 2001; Fetrati & Pak, 2020; Sarris et al., 2021)

Noting that, so far, no tests are available or exist in the literature to determine the values of the constants in the model provided in the above equation, nor to verify its form. The objective of this research was to provide insights into the nature of the λ and in particular, to present its dependence on the external conditions applied to the material. The above equation (3-8) suggests that sanding onsets gradually, till plastic strain exceeds a specific value (capturing the transient regime of erosion – not stable sand production), reaching a plateau beyond the limiting value of

the plastic strain (capturing the steady-state sand production). To minimize the variables affecting the problem to a minimum, a simplified version of the model base in Equation (3-8) has been adopted as shown in equation (3-9). In this new version, the gradual transition to the plateau has been neglected on the basis of the negligible transient regimes in hollow cylinder tests in an experimental analysis.

The new version of the proposed function, where the sand production coefficient may be simplified is shown in equation (3-9) and illustrated in figure (3.1) (Sarris et al., 2021)

$$\lambda(\varepsilon^{pl}) = \begin{cases} 0 & \text{if } \varepsilon^{pl} \leq \varepsilon_{thr}^{pl} \\ \lambda_2 & \text{if } \varepsilon^{pl} \geq \varepsilon_{thr}^{pl} \end{cases} \quad (3.9)$$

Equation (3.9) acknowledges a delay in the onset of the erosion phenomenon as plastic deformation develops and advances. As explained, the simplicity of the model agrees with the fact that the hollow cylinder rock samples are small, hence it is expected that the transient phenomena will be negligible in influencing the overall solution. However, at larger scale models, the transient phenomena may be critical; still, no detailed phenomenological analysis exists for this. In such cases, factors such as time-dependent stress redistribution, delayed particle mobilization and changing fluid pressure gradients can greatly impact erosion's initiation and development. These processes may create non-linearities and feedback mechanisms that are absent or negligible under laboratory conditions, resulting in discrepancies between model predictions and field observations. The lack of comprehensive investigation into these transient effects exposes a notable deficiency in current sand production modelling, emphasizing the necessity for focused experimental and numerical research to understand their influence. The plot of Equations 3.8 and 3.9 is shown in Figure 3.1 below.

Although the simplified formulation necessarily omits certain transitional features present in the original model of Vardoulakis et al., (1996), it serves as an effective tool for isolating the primary mechanical controls governing sand production. The reduced parameter set facilitates calibration with the limited experimental data currently available, a factor of particular importance in the absence of established testing protocols for the complete set of constitutive constants. Furthermore, the formulation offers a clear and adaptable basis for future advancements, allowing

for the integration of additional complexity once adequate experimental evidence has been obtained.

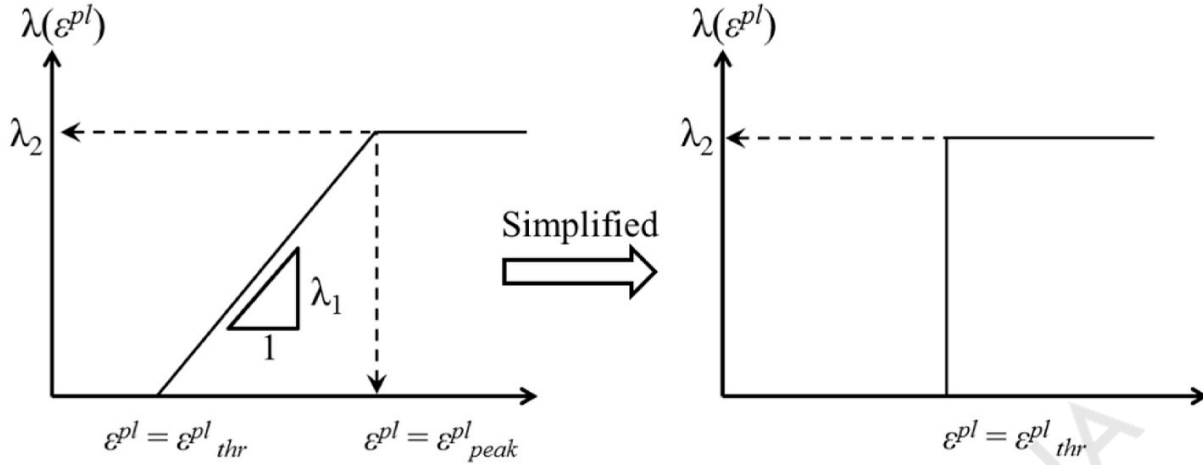


Figure 3.1 Detailed description of the sand production coefficient behavior (eq. 3-8) and (eq. 3-9) used in the simulations (Kakonitis et al., 2023a)

The time delay and the calibration constant λ_2 are fitted so that the numerically produced sand production curve compares quite well with the experimental work of Papamichos et al., (2001).

3.3 Finite Element Model for Sand Production

A finite element model was created to investigate the above hypothesis (i.e., transient phenomena are suppressed in hollow cylinder tests proving the basis for back analysis and understanding the physics of erosion coefficient λ). The model aimed to simulate the hollow cylinder tests presented in the work of Papamichos et al., (2001) for a 2-D model with an inner radius $r_w = 0.01$ m, an external radius $r_e = 0.1$ m and cylinder height $H = 0.2$ m under plane strain conditions.

The plane strain assumption used for the modelling of the hollow cylinder test was assumed to have no quantitative impact on the results. Additionally, the geometry of the rock samples allows for quarter symmetry. The model was sufficiently meshed around the cavity to resolve any minor variations on the inner face of the specimens caused by erosion, and the meshed region extends 0.02 m from the initial inner radius r_w . In addition, the model dimensions and the local region mesh refinement exclude any boundary effects (i.e. the numerically eroded surface will remain close to the inner hole rather than reaching the external boundary). The finite element model that was created, its geometry, mesh and details over the erosion area are shown in Figure 3.2. Where σ_h is

the external radial stress and Λ is the length of the induced plastic zone important parameters that will be analyzed in the following.

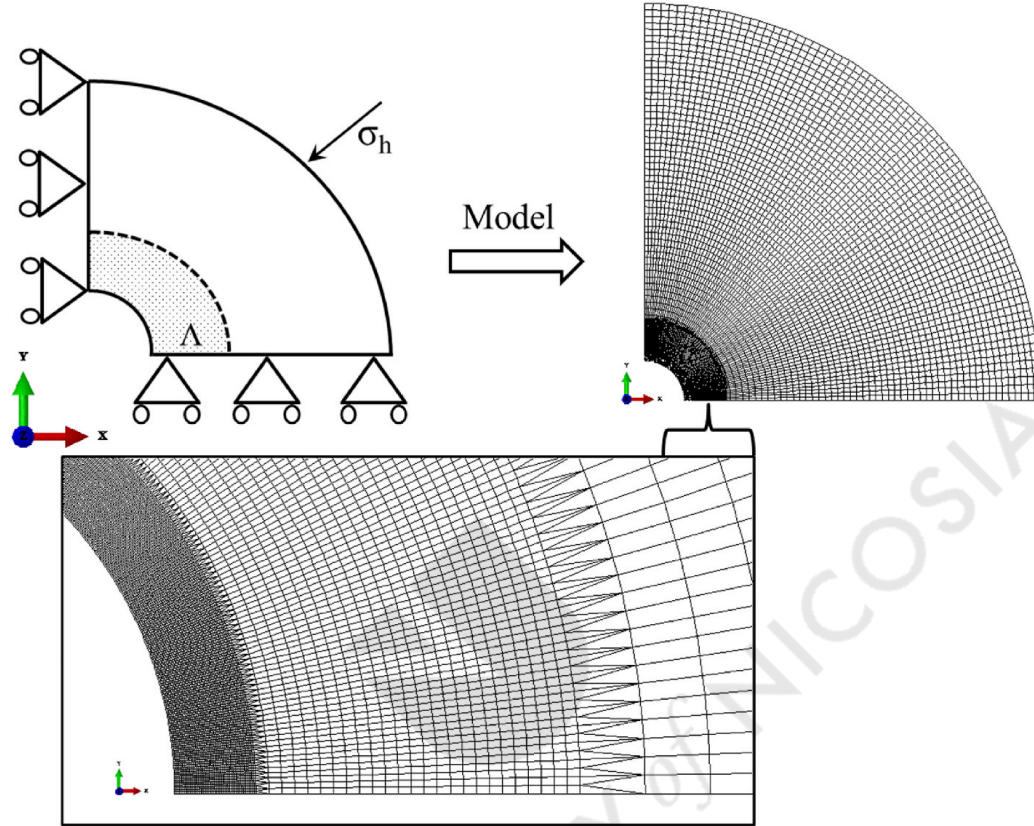


Figure 3.2 From geometry to the application of boundary conditions and the resulting finite element mesh with a detailed depiction of the very fine elements in the erosion region (Kakonitis et al., 2023a)

The finite element model was created with the commercial software Abaqus. A fully coupled model was formulated therefore, the deformations and the pore pressure degrees of freedom were the unknown variables that would be determined from the simulations. The element that used was the CPE4P, which is a 4-node bilinear displacement and pore pressure quadrilateral element. The complete mesh has 23162 nodes and 23184 elements in total. With the size of the problem at hand, it was ensured that the amount of sand produced would be accurately captured, or in the numerical language “resolved” since the erosion physical process localizes in the area of a short regime near the hole as dictated by the experimental hollow cylinder tests. It is reminded that our intention is the numerical simulations of experimental hollow cylinder tests (Kakonitis et al., 2023a).

The chosen, fully coupled, modelling method ensures accurate mechanical and hydraulic responses with sufficient resolution in the erosion zone. The quarter symmetry and biased mesh

shown in Figure 3.2, reduces computational cost while preserving unaltered the stress and pore pressure gradients near the inner cavity wall, where erosion is critical. Mesh refinement in this region and domain restriction focus the simulation on localised failure mechanisms from hollow cylinder tests, providing a solid framework for assessing the erosion model.

Based on the physics of the model described previously, when uniform radial external stress is applied, it causes stress concentration at the inner surface of the hollow cylinder, inducing local plastic strains at the inner hole, which allows the onset and advancement of the erosion process. The simulation proceeds in two steps: **(A)** At the first step, there is an initial equilibrium which allows the material to respond (i.e., reach equilibrium) from the external load, and **(B)** the second step is a dynamic stage, in which the flow rate is turned on, poroplastic deformations begin to develop, creating and enhancing the plastic zone depth Λ and then the erosion process onsets and progresses with time (Kakonitis et al., 2023a).

3.3.1 Boundary conditions of the finite element model

Both steps are operational under symmetry conditions, meaning $d_x = 0$ at the vertical axis and $d_y = 0$ at the horizontal axis (see Figure 3.2) ensuring quarter symmetry. At the symmetry planes, shear stresses are considered zero. To simulate accurately the experiment of the hollow cylinder test, on the internal boundary of the hollow cylinder, zero radial internal pressure and zero fluid pressure were applied in order to ensure that fluid flow starts from the external boundary and is directed into the internal boundary leaving the internal boundary “unsupported” and free to the erosion process. On the external boundary, a set of different external radial stresses of $\sigma_h = \{7.5, 8, 9, 10, 11\}$ MPa, were applied while also considering a constant external pressure $P_{ext} = 0.15$ MPa throughout the simulations. Moreover, for each stress level, erosion was initiated and allowed to produce sand particles for 8000 seconds as per the experimental procedure. At this point it is important to mention that we have simulated the work of Papamichos et al., (2001) which he has isolated each loading stress level. This is highly convenient for finite element simulations as each erosion stage can be modelled without the cumulative sand production curve. For the sake of the discussion, we have started the work with this modelling process and in the next chapters we have improved our understanding and modelling conditions to reflect more accurately both the confining stress and the flux boundary conditions (Kakonitis et al., 2023a).

As explained, this loading (Neumann BC) and symmetry (Dirichlet) boundary conditions setup reproduces the conditions of the experimental hollow cylinder tests, allowing the numerical results to be directly compared with observed data. Applying multiple external stress levels enables evaluation of how the erosion rate and plastic zone depth Λ respond to changes in in-situ stress. The extended simulation duration of 8000 seconds guarantees that transient effects are minimized to a point that can have zero importance (justified by the small sample dimensions) and that any steady-state erosion patterns can be detected. Based on the above conditions, the following equation provides the initial flow rate Q that passes through the hollow cylinder (Kakonitis et al., 2023a).

$$\text{Flow rate: } Q = 2\pi k_0 H \frac{P_{ext}}{\ln\left(\frac{r_e}{r_w}\right)} \quad (3.10)$$

Assuming a Kozeny-Carman coefficient $k_0 = 8.956 \times 10^{-12} \text{ m}^2$ expression (3.10) yields $Q = 0.5$ liters per min, which to the authors knowledge is quite high for experimental setups

3.3.2 Material Properties and solutions methodologies of the finite element model

The simulation of the sand production process induces considerable deformation of the material surrounding the inner aperture of the modelled hollow cylinder. This deformation is precisely documented through the refinement of the finite element mesh in regions susceptible to erosion, thereby underscoring the importance of high-quality and adequate meshing in the modelling process. In the Lagrangian-based finite element analysis, the element nodes are interconnected with the material. Consequently, any deformation caused by material modifications is manifested in the finite element mesh, as the material boundary aligns with the element boundary. This approach generates moving surfaces, and when boundary conditions are applied, excessive distortions of the finite elements can occur, leading to significant deformations. As a result, this method is deemed unsuitable for simulating the sand production process. Conversely, the Eulerian-based finite element analysis facilitates the occurrence of material alterations while maintaining the fixed position of element nodes in space. Evidently, this analysis precludes mesh distortion, rendering the application of boundary conditions particularly challenging, as the material flow must be monitored as a moving surface (Kakonitis et al., 2023a).

For purposed of this research work, the Arbitrary Lagrangian-Eulerian (ALE) finite element analysis was adopted. ALE is an adaptive meshing technique that combines the advantages of both the Lagrangian and the Eulerian analyses to simulate moving boundary/surface problems while effectively avoiding the mathematical difficulties of large strain theory and convergence issues allowing for focusing on the problem at hand rather on the development of numerical schemes. The primary benefit of the Arbitrary Lagrangian-Eulerian (ALE) method is that mesh deformation can operate independently of material alterations, rendering it highly suitable for simulating erosion processes while preserving the integrity and quality of the finite element mesh throughout the simulation, even in scenarios involving significant deformation or material loss. Consequently, the ALE methodology was deemed the most appropriate approach for the problem under consideration (Kakonitis et al., 2023a). Furthermore, a comprehensive analysis of the ALE adaptive meshing could be found in (Li et al., 2018 and Fetrati et. al., 2020). Furthermore, the ALE adaptive meshing along with adaptive constraints are available in the Abaqus finite element software.

Under natural conditions, the erosion process leads to the gradual removal of material from the exposed internal surface of the rock specimen. Conversely, in modelling contexts, this process is characterized by the adjustment of boundary conditions on the newly formed surface that results from material removal upon the fulfilment of the volumetric criterion, that was coded in Fortran as external user subroutine. It is assumed that the rock samples modelled were sandstones conforming to the Mohr-Coulomb yield criterion. Regarding the elastic component of the model, the Young's modulus and Poisson's ratio were assumed to remain constant throughout the simulations, thereby excluding Young Modulus-softening from the modelling process as per suggestions of Gravanis et al. (2015) and Stavropoulou et al., (1998) that considered Young Modulus softening in their modelling.

In relation to the plastic component, the Mohr-Coulomb criterion is characterized using the properties of cohesion, friction angle, and dilation angle. For the purpose of the research, we have considered cohesion softening of the rock through the erosion process which we hypothesize that cohesion softening influences the erosion process much harder than Young modulus softening. The cohesion-softening in this model was represented by a plastic strain dependency of the cohesion parameter, characterized by a linear softening described by equation $C = 3.7 - 50 \varepsilon^{pl}$ [MPa]. It was assumed, based on experimental investigations Shahsavari et al., (2021a), that the cohesion of the

material due to the softening behavior will not reach zero at failure. The dependence of the cohesion on ϵ^{pl} for the relevant range of values of the plastic strain is illustrated in Figure 3.3 (Kakonitis et al., 2023a):

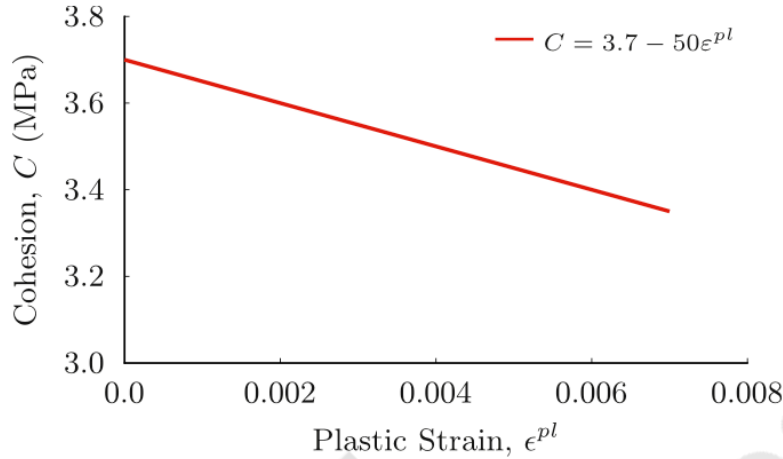


Figure 3.3 Representation of the linear cohesion softening as a function of plastic strains (Kakonitis et al., 2023a)

Concerning the porous characteristics of the rock, these are described by the dynamically evolving porosity and permeability, which emerge as part of the solution process, beginning with an initial uniform porosity value. Table 3.1 summarizes the input parameters for the simulations, including the geometry, porous rock, and permeating fluid properties. Moreover, all the physical parameters are considered from the work of Papamichos et al., (2001) which we attempt to calibrate the presented model.

The iterative numerical solution of the non-linear system of equations, outlined previously, is executed using the Newton-Raphson method. The continuity equations are integrated temporally employing a backwards Euler scheme, with all variables updated after each incremental step.

Finally, the following modelling and simulation assumptions were acknowledged (Kakonitis et al., 2023a):

- (A) The adopted hydrodynamic model describing the volumetric sand production (Equation 3-6) relates the erosion process with material yielding (plasticity) as a function of the external radial stress applied at the hollow cylinder boundary. This assumption establishes a maximum output of material yielding (e.g., worst-case scenario) for a specified external radial stress magnitude. Consequently, this assumption produces a conservative prediction

of sand production, applicable both experimentally and theoretically. We recognise that critical plastic strain criteria are also accessible for predicting sand production numerically, however, the research aimed to develop an understanding of the volumetric sand production coefficient λ and its dependencies on stress magnitude and plastic zone development, utilising as few parameters as possible (Kakonitis et al., 2023a).

Table 3.1 Standard input parameters used for the simulations adopted from Papamichos et al., (2001)

Input parameters used for the simulation	
Variable	Value
Geometric Properties	
Hollow cylinder internal radius, r_{in} [m]	0.01
Hollow cylinder external radius, r_{out} [m]	0.1
Cylinder height, H [m]	0.2
Porous rock and fluid properties	
Young modulus, E [MPa]	6750
Poisson ratio, ν [-]	0.19
Cohesion, C [MPa]	3.7
Friction angle, Φ [°]	37.4
Initial porosity, ϕ_0 [-]	0.3
Initial rock permeability, k [md]	500
Solids density, ρ_{solid} [kg/m ³]	2640
Kozeny-Carman parameter, k_o [m ²]	8.96E-12
Dynamic viscosity, μ [MPa·s]	5.00E-09

- (B) We assume that the ALE formulation sufficiently and accurately predicts sand erosion onset and production. The assumptions previously outlined serve as a convenient approximation, enabling us to investigate our hypothesis through the proposed finite element model. A comprehensive physical understanding of the sand production coefficient λ remains unavailable to date and consequently, reducing the parameters affecting the problem to a minimum, especially at experimental scales. Based on these conditions, the model could potentially be applicable, after upscaling, to more complex stress states (e.g., 2D or 3D anisotropic stress fields) and at any scale, ranging from experimental to field-scale (Kakonitis et al., 2023a).

Collectively, these assumptions establish a clear and precise framework for modelling sand production, enabling meaningful insights despite the inherent simplifications of the model.

Although simplifications were necessary due to current experimental limitations and the complex nature of erosion phenomena, the adopted approach provides a solid foundation for future refinements as additional data become available and understanding advances (Kakonitis et al., 2023a).

3.4 Results from the Finite element analysis

This section provides the results from the simulations. Figure 3.4 illustrates the relationship between the initial plastic zone depth Λ and the external radial stress. Furthermore, the blue circle markers denote the simulated stress steps of $\sigma_h = 7.5, 8, 9, 10, 11$ MPa. Conversely, the single red square marker represents the radial stress value of 7.46 MPa, which is the minimum radial stress needed to initiate plastic yielding in the material and hence erosion. It is considered important to mention that without plastic yielding, there is no sand erosion onset and production. The results illustrated in Figure 3.4 are derived after balancing the application of external radial stress steps corresponding to the studied values of stress ($\sigma_h = 7.5, 8, 9, 10, 11$ MPa) with the onset of material plastic yielding immediately prior to erosion. It is worth noting that the results are in complete agreement with the solution of the poroelastoplastic equations presented in Gravanis et al. (2015), and are indicated by orange cross markers (Kakonitis et al., 2023a).

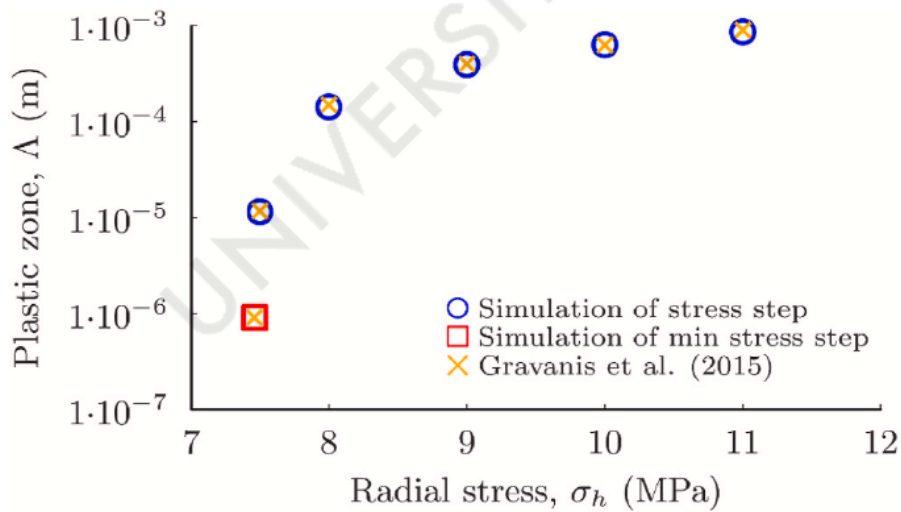


Figure 3.4 The initial plastic zone dependency on the externally applied radial stress (Kakonitis et al., 2023a)

The analysis of the results indicates that the minimum stress value examined in the experiments of Papamichos et al., (2001) (7.5 MPa) results in a markedly small plastic zone development compared to higher values. Furthermore, below the 7.5 MPa threshold, the plastic zone diminishes rapidly and disappears at a point slightly below 7.46 MPa under which erosion is not possible. The plastic yielding of the material exhibits a strong correlation with the observed erosion process behavior which confirms the hypothesis that a rock may erode after it fails locally and then the pore pressure gradients (via drag fluid forces) pick up the failed flakes and send them to the sand trap (Kakonitis et al., 2023a)

Figure 3.5 that follows, illustrates the sand production curves derived from the finite element model, corresponding to various stress σ_h values, and compares them with the experimental data of Papamichos et al., (2001) and the mathematical results of Gravanis et al. (2015). The simulations were conducted over a duration of 8.000 seconds, equivalent to 2.22 hours of sand production, with the outcomes depicted using solid lines. The experimental data from Papamichos et al., (2001) are illustrated with circular markers, while the analytical results from Gravanis et al. (2015) are indicated with dashed lines.

It should be noted, as demonstrated in Figure 3.1, that the sand production coefficient λ in the constitutive equation becomes active when the plastic equivalent strains for the Mohr-Coulomb yield criterion surpasses a certain threshold (ε_{thr}^{pl}), beyond which λ remains unchanged. This results in a certain “delay” in the initiation of erosion. Given that λ remains constant, that means that the simulated eroded mass curves do not encompass the transient phase observed in the experimental curves. This outcome is attributable to the manner in which the experiment was conducted, wherein stress was incrementally increased from 3 MPa until failure, exceeding 11 MPa. Notably, transient phenomena significantly influence the initial erosion behavior, affecting both the timing and rate of sand production onset. Therefore, accurately capturing these effects requires more complex constitutive models and time-dependent parameters, which falls outside of our scope of investigation and will be left for future investigation (Kakonitis et al., 2023a).

Through a tentative and exhausting process of trial and error, we determine after several simulations the values of the parameters ε_{thr}^{pl} and λ_2 for each external stress considered to match the simulated with the experimental curves of Papamichos et al., (2001) ensure that the simulated erosion mass curves most accurately align with the experimental data while noting that the starting

time of erosion tends to decrease as the stress values increase, which is justified by the transition time of the experiments.

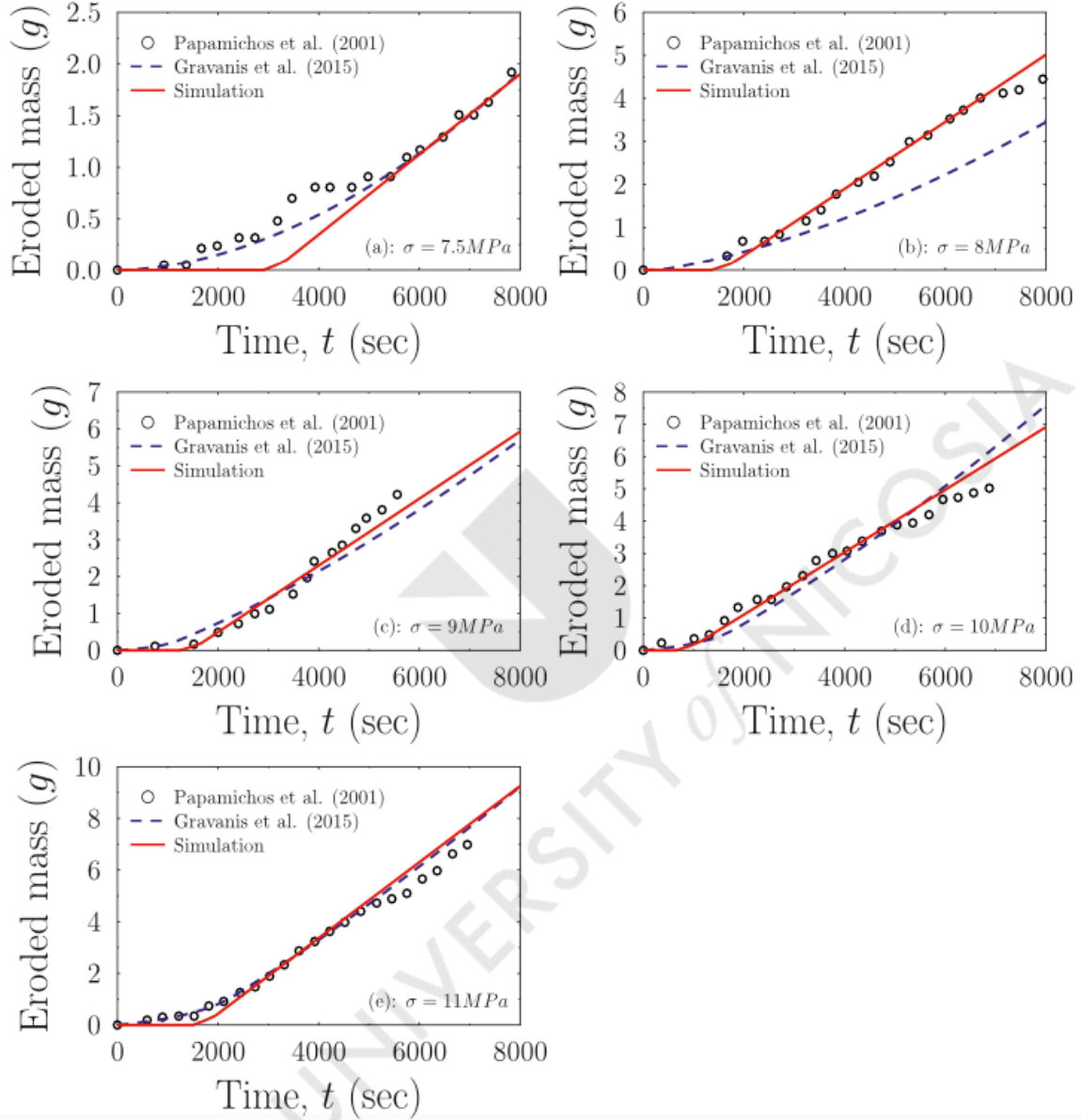


Figure 3.5 Comparison of sand production curves between (i) experimental (Papamichos et al. 2001), (ii) mathematical (Gravanis et al. 2015) and (iii) numerical for the various external radial stresses $\sigma_h = \{7.5, 8, 9, 10, 11\}$ MPa considered (Kakonitis et al., 2023a).

In the results presented within this research, this is not captured because the erosion starting time is the result of trials aimed at determining the optimum value of ε_{thr}^{pl} and λ_2 which do not appear to have this trend due to the lack of the transient physics in the simulations. Furthermore, the

theoretical sand production curves of Gravanis et al. (2015), were included although they are obtained with a different model for the sand production coefficient. Additionally, from the point of view of the material, an elastoplastic behavior with hardening could be adopted to reduce the discrepancy in the transient regime between the experiments and the simulations. However, this would result in increasing the number of parameters influencing the estimation of λ_2 while the research intended to produce a rough estimation for the sand production coefficient. This is consistent with the choice to keep the input parameters at a minimum, which was also adopted in the hydrodynamic model (Kakonitis et al., 2023a).

Analysis of the simulation results indicates that the numerical model accurately reflects the experimental outcomes across all considered radial stress values. The stability of the sand production factor suggests that the erosion mass rate remains nearly constant throughout the simulations. However, it is important to recognize that during the initial stages, a transient regime occurs, linked to the middle term of equation (3.8), which we have reduced in the model by implementing a Fortran subroutine alongside with the erosion constitutive law (equation 3.6). This fact, along with the issue of the transient parts of the experimental curves, generates some differences between the experiments and the simulations, which are quite visible in the 7.5 MPa case. From a physical point of view, after a certain period of sand production, it could be observed that the surface area of the hollow cylinder enlarges, indicating a reduction in the frictional forces between solid particles (see e.g Figure 3.9). This hole enlargement results in an abrupt drop in sand production (late times in the experimental results), as evidenced in the experimental curves under external levels $\sigma_h = 7.5, 8, \text{ and } 10 \text{ MPa}$, which is associated with catastrophic failure of the hollow cylinders Papamichos et al., (2001).

The current model does not account for this late-stage effect, commonly referred to as friction softening, and this limitation will be addressed in future research. It is important to note that the identified limitations, particularly those related to transient behavior and late-stage friction softening, are not exclusive to this study. Similar discrepancies have been documented in the existing literature when simplified constitutive laws are employed for sand production modelling, such as those lacking explicit consideration of progressive particle detachment or evolving contact mechanics (Vardoulakis et. al., (1998) and Papamichos et. al., (2001)). Furthermore, Figure 3.5 demonstrates a better agreement between the simulation outcomes and other results at higher stresses. The reason for this observation is that, at lower stress levels, the erosion process advances

gradually, resulting in extended transient regimes until stabilized sand production can be observed. Conversely, at higher stress values, the erosion process proceeds more rapidly, diminishing the transient regime and leading to earlier stabilization of sand production (Kakonitis et al., 2023a). Figure 3.6, shows the tangential and radial stresses as functions of distance under an external stress of 11 MPa, corresponding to early and late time respectively. The results are illustrated using dashed-red lines for the simulations and continuous-blue lines for the mathematical model proposed by Gravanis et al. (2015). The graphs demonstrate the influence of softening on the stress fields over time (Kakonitis et al., 2023a).

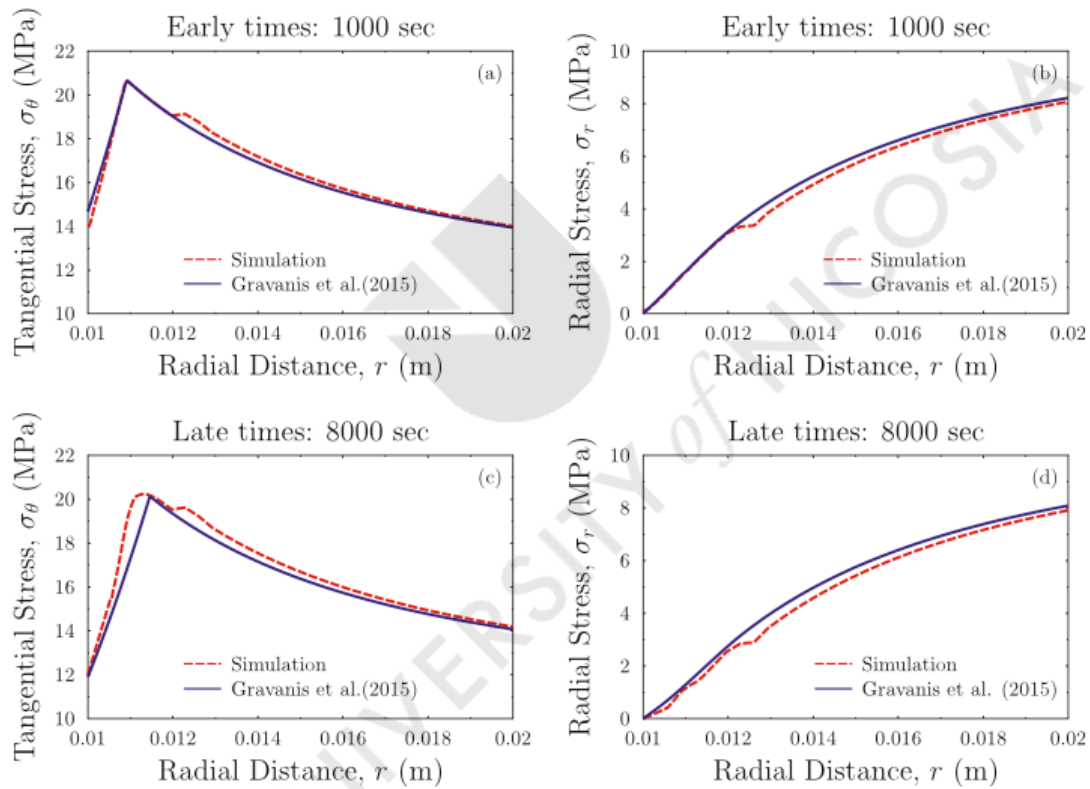


Figure 3.6 Comparison between the simulations and the solution of Gravanis et al. (2015). (a) Tangential stresses and (b) radial stresses for early times 1000 sec (0.27 hours), (c) tangential stresses and (d) radial stresses for late times 8000 sec (2.22 hours) (Kakonitis et al., 2023a).

As anticipated, with the advancement of time, tangential stresses σ_θ drop on the inner surface of the hollow cylinders, leading to a corresponding decrease in the peak values at the boundary of the plastic with the elastic zone. In other words, the tangential stress presents a maximum at the elastic-plastic boundary and rapidly drops as time progresses denoting material softening prior to failure.

Furthermore, as erosion advances, the peak values shift away from the axis of the hollow cylinder, signifying the ongoing progression of the erosion process.

The radial stress field configuration σ_r also shifts away from the cylinder axis as more and more material is eroded. Although the mathematical model employs a distinctly different mechanism of weakening (with cohesion depending on the evolving porosity), the overall impact of cohesion weakening on the stress fields remains nearly identical, for nearly the same sand production. The analytical model developed by Gravanis et al. (2015) employs a methodology fundamentally different from that of the proposed finite element model, and it is anticipated to exhibit enhanced performance. Specifically, Gravanis et al. (2015), model (i) accounts for the degradation of both cohesion and Young's modulus of the material in question, and (ii) the erosion coefficient is characterized by a three-parameter model, providing greater adaptability within the transient regime which we ignore in the modeling. However, the analytical model demonstrated that deterioration of the Young's modulus does not produce significant effects. Furthermore, the erosion strength, derived from material cohesion and the erosion coefficient, presents an elegant approach for finite element modelling.

Despite their differences, the final outcome of the sand production prediction aligns reasonably well. However, due to the simplification in Equation 3-9, discrepancies are observed in the transition region of the sand production curves. Additionally, as demonstrated in Figure 3.6, the proposed model exhibits an advantage in the comparison of tangential and radial stresses during both early and later stages. Considering that the finite element model is compared with experimental data and an analytical solution, the research could be expanded to include more complex stress states and in more complex geometries, beyond the hollow cylinder tests for which it was originally developed, where the analytical solution is inapplicable. This comparison demonstrates that simplified numerical formulations, when appropriately calibrated, can attain predictive accuracy comparable to that of more elaborate analytical frameworks, even when developed under varying modelling philosophies.

The foregoing analysis facilitates the investigation of the dependency of the sand production coefficient, as determined through inverse analysis, on the magnitude of external stress. This model addresses a significant gap in the current understanding of the hydrodynamic model presented in Equation 3-6, which describes the production of solid mass. The optimal values of the sand production coefficient are obtained through back analysis using an appropriate power law function.

As emerges from the back analysis we propose Equation 3.11 to describe the erosion coefficient (Kakonitis et al., 2023a).

$$\boxed{\text{Erosion coefficient: } \lambda_2 = \lambda_{2,0} \left(\frac{\sigma - \sigma_{\min}}{\sigma_{\min}} \right)^n} \quad (3.11)$$

where: $\lambda_{2,0}$ is a scale for the sand production coefficient (units inverse length), σ is the stress level at the external boundary (i.e. in the model, it is σ_h) while in general σ denotes the minimum in-situ stress, $\sigma_{\min}$ is the least stress level to cause plastic yielding of the rock and n is an exponent.

We have determined the following values: $\sigma_{\min}=7.49$ MPa, $n = 0.1641$, $\lambda_{2,0} = 0.0912 m^{-1}$ from fitting equation (3.11) on the simulations results after benchmarking with the experiments of Papamichos et al., (2001).

The usefulness of the proposed model (Equation 3.11) lies in the fact that the sand production coefficient λ , as referenced in the constitutive law (Equation 3.6), is influenced by various factors and parameters associated with the erosion process. In this context, the investigation was exclusively focused on the dependency related to the external stress field. Furthermore, research indicates that this model $\lambda_{2,0}$ remains consistent across all external stress levels. The value of $\sigma_{\min}$ can be predicted analytically, and parameter n is an exponent that can be determined with relative ease as a fitting parameter (Kakonitis et al., 2023a).

To predict $\sigma_{\min}$, the results from Gravanis et al. (2015) were employed to derive the Equation. 3.11. Ignoring the effect of pressure, the stress field solution on the elastic side, i.e. $r > r_{in} + \Lambda$, is (Kakonitis et al., 2023a):

$$\begin{aligned} \sigma_r &= \sigma_{out} + C_2 \left[\frac{1}{r_{out}^2} - \frac{1}{r^2} \right] \\ \sigma_\theta &= \sigma_{out} + C_2 \left[\frac{1}{r_{out}^2} + \frac{1}{r^2} \right] \end{aligned} \quad (3.12)$$

where: C_2 is an integration constant to be determined by the continuity conditions across the plastic zone boundary, σ_{out} is the external stress on the outer boundary of the rock sample, r_{out} is the outer radius, and r_{in} is the inner radius.

The fields σ_r and σ_θ are the radial and tangential stresses, respectively. In the plastic zone side, i.e. $r < r_{in} + \Lambda$, the stress field is given by (Kakonitis et al., 2023a):

$$\begin{aligned}\sigma_r &= \sigma_{in} \frac{r^{K-1}}{r_{in}^{K-1}} - \frac{S_0}{K-1} \left[1 - \frac{r^{K-1}}{r_{in}^{K-1}} \right] \\ \sigma_\theta &= S_0 + K\sigma_r\end{aligned}\quad (3.13)$$

where: σ_{in} is the inner radius stress, and

$$\begin{aligned}K &= \tan^2 \left(\frac{\pi}{4} + \frac{\Phi}{2} \right) \\ S_0 &= 2C\sqrt{K}\end{aligned}\quad (3.14)$$

where: C is material cohesion [MPa], Φ is the material friction angle [$^\circ$], S_0 is the uniaxial compressive strength [MPa] (Kakonitis et al., 2023a).

The continuity condition across the plastic zone boundary reads (Kakonitis et al., 2023a):

$$\begin{aligned}\sigma_{r\text{elastic}}(r_{in} + \Lambda) &= \sigma_{r\text{plastic}}(r_{in} + \Lambda) \\ \sigma_{\theta\text{elastic}}(r_{in} + \Lambda) &= \sigma_{\theta\text{plastic}}(r_{in} + \Lambda)\end{aligned}\quad (3.15)$$

The first condition provides a value for the integration constant C_2 . By setting the plastic zone depth $\Lambda=0$, and considering the pertinent case $\sigma_{in}=0$, one easily finds (Kakonitis et al., 2023a):

$$\sigma_{out} = \left(1 - \frac{r_{in}^2}{r_{out}^2} \right) C \tan \left(\frac{\pi}{4} + \frac{\Phi}{2} \right) \quad (3.16)$$

which we identify as an estimate for the $\sigma_{\min}$ parameter of the model in equation 11. By doing so, the number of unknowns required to determine the value of the corrosion coefficient λ was reduced, which is usually viewed as a *complex 3-parameter function* with many dependencies.

One might criticize that $\lambda_{2,0}$, which has the same dimensions [1/m], still needs to be determined. This observation is accurate, however, the value within the proposed model $\lambda_{2,0}$ will be constant across all stress levels and does not vary in response to changes in the stress field. Consequently, the model estimates the sand production coefficient λ solely as a function of external stress, which is significantly simpler to determine than the three-parameter function. It is important to emphasize that the research does not claim that the sand production coefficient λ depends exclusively on the external stress level (Kakonitis et al., 2023a).

By adopting a constant λ_2 , one benefit that arises is the reduction in calibration uncertainty, as fewer parameters require independent measurement or back-calculation from experimental data. This not only facilitates model validation against a wider range of laboratory results but also improves its transferability to field-scale problems, where sparse or indirect measurements often constrain parameter estimation. Further investigation is required to assess whether the constant λ_2 assumption holds under varying operational conditions, such as changes in fluid properties, fluid flow, temperature, or complex in-situ stress fields. Furthermore, it is noteworthy that the empirical expression proposed (Equation 3.11) was derived from the hollow cylinder test, incorporating all its simplifications. The model described by Equation 3.11 maintains certain general characteristics that could be applicable to real-world field applications. This simplification, although based on controlled experimental conditions, provides a practical pathway for calibrating predictive models in field scenarios where data availability is limited (Kakonitis et al., 2023a).

Figure 3.7 illustrates the sand production coefficient λ_2 as a function of the radial external stresses used to generate the numerical sand production curves depicted in Figure 3.5. The estimated sand production coefficient λ_2 is shown with a black circle marker. As previously stated, the hydro-mechanical model is predicated on the hypothesis that erosion initiation is associated with the critical plastic deformation criterion subsequent to plastic yielding, a critical assumption of the finite element model. Figure 3.7 illustrates that the yield zone for this material manifests when the external stress surpasses a threshold slightly below 7.5 MPa (at 7.46 MPa, the depth of the plastic zone measures a micrometer).

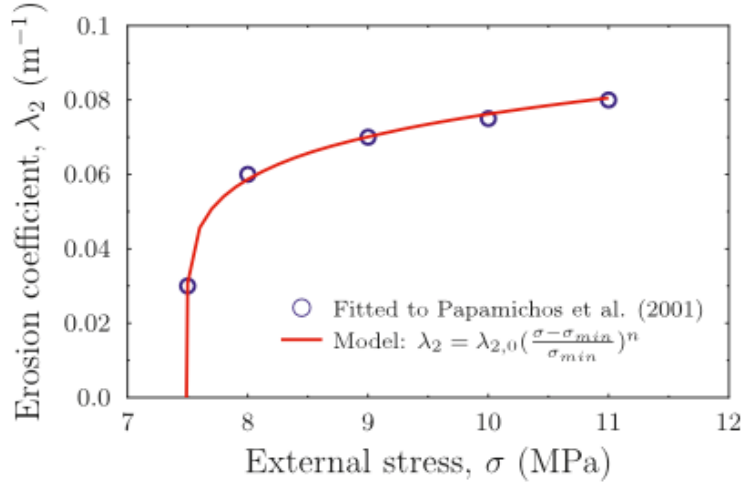


Figure 3.7 The proposed expression to describe the dependence of the coefficient λ_2 on external stress (Kakonitis et al., 2023a)

This is confirmed by the estimated value of the $\sigma_{\min}$ parameter in the power law model outlined in Equation 3.11, which is indeed slightly below 7.5 MPa. It is important to note that this outcome is derived exclusively from the observed trend of the dependence of λ_2 on the external stress (Kakonitis et al., 2023a). Therefore, the parameter $\sigma_{\min}$ can be theoretically estimated based on static poroelastoplastic solutions Gravanis et al. (2015) by identifying the maximum external stress that does not cause plastic yielding. Indeed, by neglecting the influence of pressure, an analytical expression for the parameter $\sigma_{\min}$ can be derived, as represented by the following relation:

$$\sigma_{\min} = \left(1 - \frac{r_{\text{in}}^2}{r_{\text{out}}^2} \right) C \tan \left(\frac{\pi}{4} + \frac{\Phi}{2} \right) \quad (3.17)$$

For the specific material and the conditions of the current problem the estimation of $\sigma_{\min}=7.41$ MPa which is 1% lower than the estimated value and is very close to the best fit result of 7.49 MPa. Taking r_{out} to be high, this expression gives $\sigma_{\min}=7.49$ MPa (Kakonitis et al., 2023a).

The analysis revealed a correlation between the sand erosion coefficient λ_2 and the depth of the plastic zone, Λ . The simulations indicated that the external stress level, σ_h , impacts both the erosion coefficient λ_2 and the extent of the plastic zone. A new model has been proposed to describe the dependence of the sand production coefficient λ on the depth of the plastic zone Λ . Unlike the model presented in Equation 3.11, this model incorporates not only the material properties and

external stress but also the influence of pressure drawdown. Mathematically, the model is represented by the following relation (Kakonitis et al., 2023a):

$$\text{Erosion coefficient: } \lambda_2 = \beta \Lambda^\gamma \quad (3.18)$$

where: β is a dimensionful proportionality constant and γ is an exponent. With the optimal λ_2 values estimated through back analysis, namely $\beta = 0.4592$ and $\gamma = 0.2379$, where λ_2 and Λ are expressed in meters (Kakonitis et al., 2023a).

Figure 3.8 illustrates the model curve in a continuous red line and the estimated λ_2 in black circle markers.

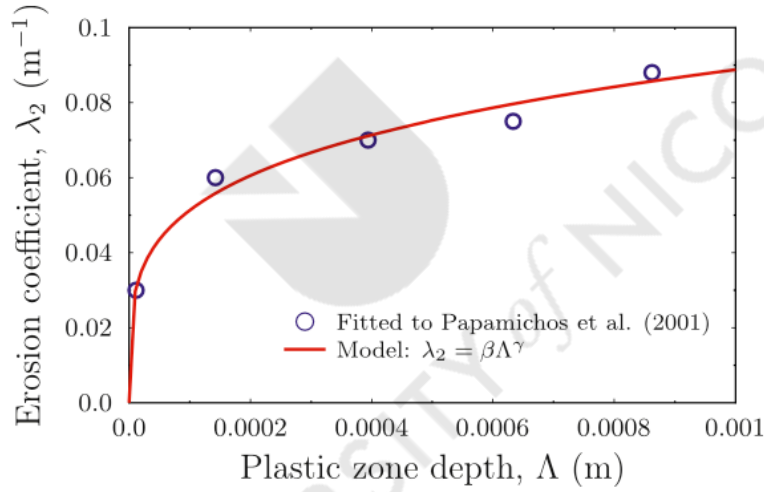


Figure 3.8 Proposed expression to describe the dependence of the coefficient λ_2 on the plastic zone depth Λ (Kakonitis et al., 2023a)

It is important to note the manner in which the plastic zone Λ and its shape were defined. As previously mentioned, plastic yielding follows the Mohr-Coulomb criterion.

$$\varepsilon^{pl} = \int \frac{1}{C} \sigma : d\varepsilon^{pl} \quad (3.19)$$

where: ε^{pl} are the plastic strain increments, σ is the stress change in the material and C is the material cohesion. Plastic zones are developed as a function of stress level and continue to expand

and develop as the material undergoes erosion. The following relation provides the calculation method for the plastic equivalent stresses according to the Mohr-Coulomb criterion (Abaqus, 2020).

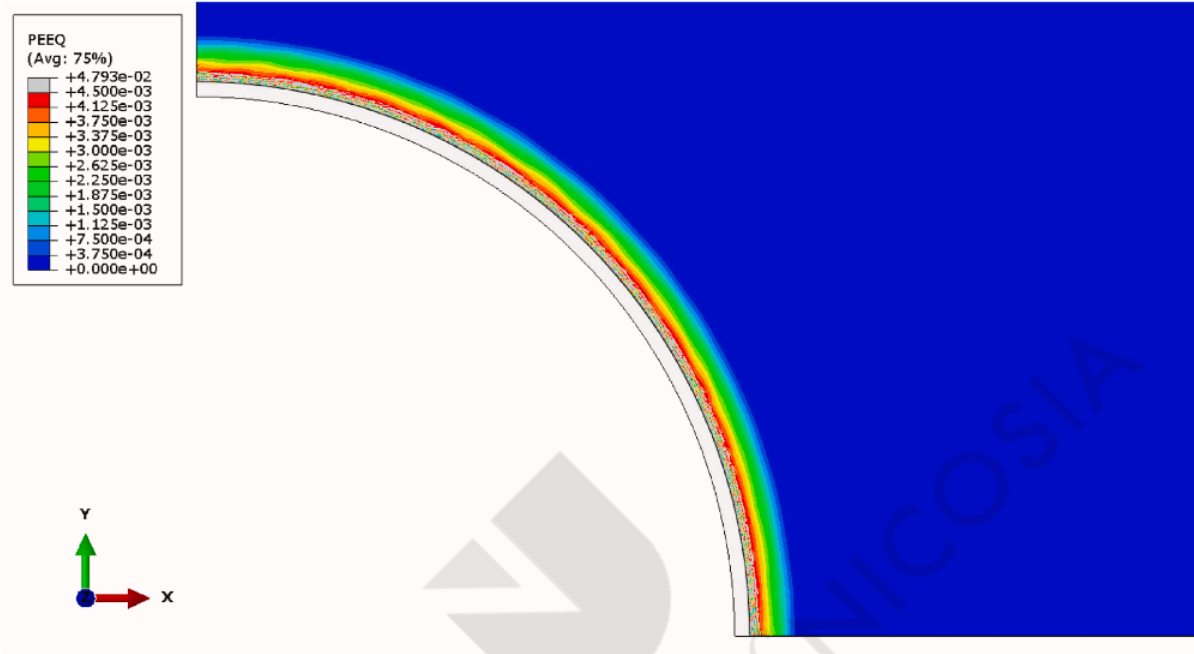


Figure 3.9 Numerical solution for the equivalent plastic strains showing the eroded surface, the associated development of plastic zone A, and the shape of the plastic zone during evolution. Simulation time is 8000 sec (2.22 hours) (Kakonitis et al., 2023a)

Given that the model is designed as a hollow cylinder (see Figure 3.2), the external stress boundary condition is uniformly applied in the radial direction, facilitating the development of a symmetric plastic zone. This geometric simplification allows the generation of sand particles symmetrically from the inner surface of the cylinder, forming a cylindrical erosion surface. Figure 3.9 illustrates the development of a plastic zone (Equation 3.19) around the inner cavity of the hollow cylinder. Starting from the inner surface, the eroded material is illustrated as a grey region, indicating that the eroded surface has advanced from its original position. As the erosion process continues, additional material becomes available for plastic yielding and subsequently erosion. The region of plastic yielding is represented by the colored area on the map. Finally, the remaining intact material that has not undergone yielding is shown as the blue area. Therefore, owing to the symmetrical

geometry and the radial boundary condition, the shape of the plastic zone can be easily defined (Kakonitis et al., 2023a).

The simulation time for Figure 3.9 and for the presented results in this chapter, is 8000 seconds, equivalent to 2.22 hours. It is important to observe that, under different geometries and stress conditions, the erosion surface and the determination of the shape of the plastic zone may not exhibit symmetry. In such cases, the final eroded form and the shape of the plastic zone will inherently result from the Arbitrary Lagrangian-Eulerian (ALE) solution, which offers the actual erosion profile, plastic zone shape and ensures numerical stability.

3.5 Chapter Key Findings

A portion of the research delineated in this chapter, involved revisiting the issue of sand production in hollow cylinder tests within the framework of hydrodynamic modelling of the erosion process, with the objective of establishing empirical relationships for the sand production coefficient as functions of external conditions, which are presently lacking. These relationships are based on the physical assumptions of the hydrodynamic erosion models, which correlate the erosion process to the effect of plasticity, given that the latter results from external stress. For this reason, empirical values for the sand production coefficient λ_2 were determined by best fitting the experimental data of Papamichos et al., (2001) with numerical simulations. The results show that λ_2 has a fairly smooth dependence on external conditions. Two models were proposed to describe this dependence:

- A. The initial model presumes λ_2 as a function of external stress and involves three parameters: a scale parameter, an exponent, and a stress value at which λ_2 becomes zero. Notably, this stress value closely corresponds to the minimal external stress required to initiate plastic yielding. This observation further clarifies the proposed linkage between the erosion process and plasticity. It is noteworthy that the minimal stress for plastic yielding can be derived theoretically, and the corresponding results have been provided, thereby effectively reducing the number of parameters in the model by one.
- B. The second model proposed correlates λ_2 with the depth of the plastic zone Λ via a power law expression. The potential importance of this model resides in the fact that the depth of the plastic zone encompasses the effect of pressure drawdown, although this effect is relatively minor in the case under consideration.

The propositions presented in this chapter aimed to enhance hydrodynamic erosion models to clarify the nature of the purely phenomenological λ_2 coefficient. Additional and thorough comparisons with experimental data are necessary to determine the validity range of the proposed models, particularly the stress-dependent variant, and to explore the potential universality or the dependencies of scale parameters and exponents. In particular, validation under multi-axial stress states, variable fluid properties, and heterogeneous material structures would be essential to evaluate the robustness of the proposed correlations across a wider range of operational conditions scenarios. This remains a prospective subject for future investigation.

The proposed methodology for simulations potentially serves as a valuable instrument for the characterization of geological media regarding their erosivity. Furthermore, the estimation of volumetric erosion law parameters can be employed for predictive modelling purposes. From a wider perspective, the proposed formulations help bridge the gap between phenomenological sand production coefficients and physically based constitutive descriptions. By providing a link between measurable field or laboratory parameters and erosion behavior, the methodology can support both theoretical advancements in coupled geomechanics-hydrodynamics modelling and practical workflows in reservoir engineering.

CHAPTER 4-A SEMI-ANALYTICAL MODEL FOR SAND PRODUCTION

4.1 Preamble

This chapter introduces a semi-analytical hydro-mechanical framework for modelling sand production from hollow cylinder tests with small to negligible induced pore pressure. The methodology considers the erosion process as a direct result of the plastic zone development, governed by hydrodynamic principles and coupled with material degradation based on the Mohr–Coulomb criterion. By reducing the problem to the two limiting key variables, (i) the mean porosity and (ii) the depth of the plastic zone, the framework offers a straightforward yet effective tool for analyzing erosion dynamics.

This study is restricted to laboratory conditions, where pressure drawdown is insignificant relative to the externally applied confining stress, and investigates the influence of various hydrodynamic and degradation laws. Sensitivity analyses demonstrate how changes in these laws affect erosion rates and the advancement of plastic yielding in the material. The framework is subsequently validated against experimental data and extended by incorporating a stress-dependent erosion coefficient λ , which enhances consistency with observed sand production curves. Overall, the model provides a practical foundation for calibrating erosion behavior across different rock types while maintaining adaptability for future enhancements for incorporating cases where pressure effects are significant.

4.2 Mathematical model

The mathematical model proposed in this study seeks to minimize the complexity of hydrodynamic erosion models, aiming to simplify as much as possible the effects of the mechanisms involved, which namely include (i) material plastic yielding incorporating degradation laws and (ii) hydrodynamic erosion through constitutive laws derived from filtration theories. Furthermore, the impacts of degradation and hydrodynamic erosion models were examined in greater detail and broader scope through an appropriate set of models.

The modelling of the physical problem of erosion encompasses two primary processes:
(a) the behavior of the rock under poro-mechanical conditions and

(b) the hydrodynamic erosion process taking place at the inner surface of the hollow cylinder rock specimen.

The model developed in this chapter is based on the seminal sanding model developed by Vardoulakis et al., (1996) which was later expanded by Papamichos et al., (2001) and Gravanis et. al., (2015), who the latter introduced an innovative semi-analytical formulation addressing the erosion problem as applied to the hollow cylinder test.

As explained earlier, our focus is devoted to the simplified geometry of the standard hollow cylinder test, which facilitates a conceptual reassessment of the governing equations of the problem. Assuming that, the sample is (i) homogeneous and (ii) fully saturated by a single-phase fluid, the constructed model reduces the necessary dynamical variables describing the evolution of erosion to the depth of the plastic zone and the mean porosity within this zone. These two variables depend solely on time. Accordingly, the governing equations of the model are formulated as a system of ordinary differential equations, derived through the following mechanisms:

- (i) The depth of the plastic zone is determined in a quasi-static manner based on the continuity of the stress field across the boundary of the plastic zone, as proposed initially by Gravanis et. al., (2015). This approach enables us to assume that the influence of pressure can be reduced to cases where the gradient is so small to be considered negligible. The rationale for this assumption is grounded in observations from certain experimental cases presented by Papamichos et. al., (2001), where the fluid pressure is nearly two orders of magnitude less than the applied stress.
- (ii) The plastic zone develops and propagates according to a degradation law that characterizes the softening of the rock, expressed through the porosity-dependent cohesion C and the friction angle Φ . It should be noted that, within the scope of this research, two distinct laws were employed.
- (iii) The dynamics of the mean porosity are derived from the hydrodynamic erosion model developed by Vardoulakis et. al., (1996), adopted by Papamichos et. al., (2001) and further developed by Gravanis et al., (2015).

The experimental setup consists of a hollow cylindrical specimen with an inner radius r_{in} , an outer radius r_{out} , and a height H . An external uniform radial stress σ_{out} is applied to the outer surface boundary while the inner surface is left without the application of stress (e.g $\sigma_{in}=0$). The external uniform radial stress σ_{out} induces plastic yielding within a zone of thickness Λ , as shown in Figure

4.1. Throughout the test, a constant and radially uniform flow rate Q , is maintained through the application of an external pressure (see equation 3.10).

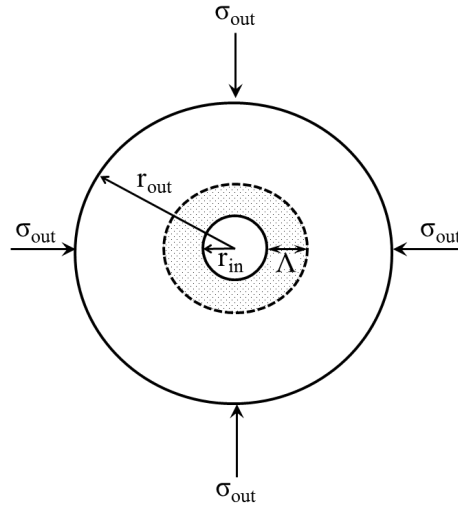


Figure 4.1 Schematic representation of the hollow cylinder test (Kakonitis et al., 2023b)

Erosion onsets and sand is produced exclusively from the plastic region, thus the Plastic zone depth Λ is of paramount importance in the analysis. This is one of our major assumptions. Although sanding may not be directly associated with rock yielding but rather with the magnitude of strain, this simplifying assumption has been adopted and seems to be scientifically valid and an alternative to the criterion of strain magnitude (Kakonitis et al., 2023b).

Figure 4.2 depicts a schematic representation focusing on the area of the hollow cylinder where an interface exists between the affected and unaffected regions. Plastic region Λ , is characterized by its mean porosity $\bar{\phi}$, and the non-yielded elastic region, which maintains a consistent initial porosity ϕ_0 at time t . As erosion advances, plastic yielding progresses uniformly in all directions (being cylindrical in shape and by the application of a radial stress, this enforces a 1-D analysis). At time $t+dt$, the plastic zone will have expanded to a thickness of $\Lambda+d\Lambda$ with an updated mean porosity $\bar{\phi} + d\bar{\phi}$.

The specific form of the porosity field (e.g., the Kozeny-Karman equation) is not of significance in our formulation. The mean porosity $\bar{\phi}$ within the plastic region and its thickness Λ will serve as the fundamental dynamical variables in our approach.

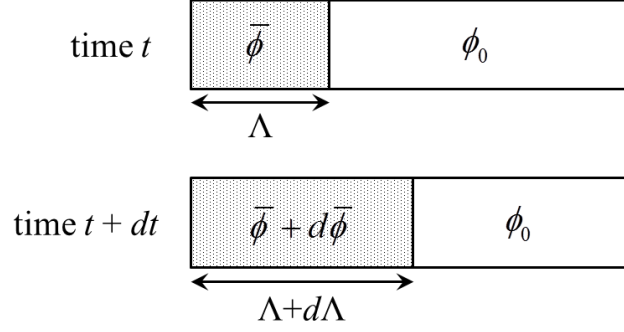


Figure 4.2 Schematic representation of the erosion interface, affected and unaffected regions (Kakonitis et al., 2023b).

The proposed semi-analytical model leverages the hollow cylinder assumption under axisymmetric conditions and comprises from three components:

- (i) an elastoplastic function of the material (e.g., Mohr-Coulomb),
- (ii) a constitutive law governing the degradation of the material (causing plastic yielding), and
- (iii) a hydrodynamic law governing the erosion process (causing erosion and sand production).

Under axisymmetric conditions, the stress field equations within the elastic region where $r > R = r_{in} + \Lambda$ (undamaged region) according to Gravanis et. al., (2015), after ignoring pressure-dependent terms, simplify to:

$$\sigma_r = \sigma_{out} + C_2 \left[\frac{1}{r_{out}^2} - \frac{1}{r^2} \right] \quad (4.1)$$

$$\sigma_\theta = \sigma_{out} + C_2 \left[\frac{1}{r_{out}^2} + \frac{1}{r^2} \right] \quad (4.2)$$

where: σ_r is the radial component of the stress field, σ_θ is the tangential component of the stress field, and C_2 is an integration constant that is determined by the boundary conditions. In the plastic region $r < R = r_{in} + \Lambda$ (damage region) the radial stress field is given by Gravanis et. al., (2015) as:

$$\sigma_r = -\frac{S_0}{k-1} \left[1 - \frac{r^{k-1}}{r_{in}^{k-1}} \right] \quad (4.3)$$

Here, S_0 represents the uniaxial compressive strength, and the parameter k is defined in terms of cohesion C and the friction angle Φ by the following equations:

$$k = \tan^2\left(\frac{\pi}{4} + \frac{\Phi}{2}\right), \quad S_0 = 2C\sqrt{k} \quad (4.4)$$

The tangential component of the stress field is determined using the Coulomb failure criterion:

$$\sigma_\theta = S_0 + k\sigma_r \quad (4.5)$$

Based on these equations, the boundary of the plastic zone, occurring at a radius $r=R$, is defined by the continuity of the stress field as:

$$\sigma_{\theta\text{elastic}}(R) = \sigma_{\theta\text{plastic}}(R), \quad \sigma_{r\text{elastic}}(R) = \sigma_{r\text{plastic}}(R) \quad (4.6)$$

One of these equations determines the constant C_2 , while the other sets the equilibrium condition, which defines R in terms of the material parameters and boundary conditions.

The equilibrium constraint is expressed as a surface within the space of the variables R , C , Φ , σ_{out} defined by:

$$F(R, C, \Phi) = 0 \quad (4.7)$$

where

$$F(R, C, \Phi) = (k-1)S_0 r_{\text{in}} R^{k+1} + (k+1)S_0 r_{\text{out}}^2 r_{\text{in}} R^{k-1} - 2(S_0 + (k-1)\sigma_{\text{out}}) r_{\text{out}}^2 r_{\text{in}}^k \quad (4.8)$$

Changing the values of C and Φ by degradation and, in general, varying also σ_{out} , moves R along that surface according to:

$$dF = \frac{\partial F}{\partial R} dR + \frac{\partial F}{\partial C} dC + \frac{\partial F}{\partial \Phi} d\Phi = 0 \quad (4.9)$$

The derivatives in this expression produce complex terms, which will be omitted for simplicity. Thus, the mechanical formulation of the problem is complete.

Material degradation is represented by permitting the cohesion and friction angle to be functions of porosity which in this instance, is characterized by the mean porosity.

$$C = C(\bar{\phi}), \quad \Phi = \Phi(\bar{\phi}) \quad (4.10)$$

For now, this dependence is kept general.

The hydrodynamic erosion law builds on the model introduced by Vardoulakis et. al., (1996) further refined by Papamichos et. al., (2001) and Gravanis et. al., (2015). We incorporate the mean porosity within the plastic zone, which is introduced and determined through proper integration of the porosity field.

$$\bar{\phi} = \frac{1}{\Lambda} \int_{r_{in}}^R \phi dr = \frac{1}{\Lambda} \int_{r_{in}}^{r_{in}+\Lambda} \phi dr \quad (4.11)$$

The constitutive equation introduced by Vardoulakis et. al., (1996) is presented as:

$$\frac{\partial \phi}{\partial t} = \lambda f(\phi) q \quad (4.12)$$

where λ [m^{-1}] denotes the sand production coefficient, assumed to be constant (otherwise the coupling is dynamical and cannot be resolved in a semi-analytical form); q represents the Darcy flux of the fluid; and f is a function of porosity. In previous formulations, f was defined as $f(\phi)=1-\phi$, although it may assume various forms, provided that $f(1)=0$, ensuring that porosity ceases to increase at the value of 1 with the physical meaning of sand produced material. The effects of critical drawdown will not be considered in this formulation. However, they can be incorporated into this equation by replacing the Darcy flux q with $q-q_{cr}$, where q_{cr} is a constant representing critical drawdown (Younessi et al., 2013).

As it can be easily included in the equations of the proposed framework, continuity implies that the radial Darcy flux q is expressed as (Gravanis et al., 2015):

$$q = \frac{Q}{2\pi Hr} \quad (4.13)$$

According to Equation (4.11), the time-variation of the mean porosity is expressed as:

$$\begin{aligned} \frac{d\bar{\phi}}{dt} &= \frac{d}{dt} \left(\frac{1}{\Lambda} \int_{r_{in}}^{r_{in}+\Lambda} \phi dr \right) = -\frac{1}{\Lambda^2} \frac{d\Lambda}{dt} \int_{r_{in}}^{r_{in}+\Lambda} \phi dr + \frac{1}{\Lambda} \phi(r=r_{in}+\Lambda) \frac{d\Lambda}{dt} + \frac{1}{\Lambda} \int_{r_{in}}^{r_{in}+\Lambda} \frac{\partial \phi}{\partial t} dr \\ &= -\frac{1}{\Lambda} \frac{d\Lambda}{dt} \bar{\phi} + \frac{1}{\Lambda} \phi_0 \frac{d\Lambda}{dt} + \frac{1}{\Lambda} \int_{r_{in}}^{r_{in}+\Lambda} \frac{\partial \phi}{\partial t} dr \end{aligned} \quad (4.14)$$

where we employed the Leibniz rule to differentiate the integral with time-dependent end-points as:

$$\frac{d}{dt} \int_{r_1(t)}^{r_2(t)} f(r,t) dr = f(r_2(t),t) \frac{dr_2(t)}{dt} - f(r_1(t),t) \frac{dr_1(t)}{dt} + \int_{r_1(t)}^{r_2(t)} \frac{\partial f(r,t)}{\partial t} dr \quad (4.15)$$

For integrating a time-dependent function f between moving end points r_1 and r_2 , we also considered the fact $\phi = \phi_0$ that at $r=R=r_{in}+\Lambda$ and utilized Equation (4.11) in the first term of (4.14). Consequently, we arrive at Equation (4.16):

$$d\bar{\phi} = -\frac{d\Lambda}{\Lambda} \bar{\phi} + \frac{d\Lambda}{\Lambda} \phi_0 + dt \frac{1}{\Lambda} \int_{r_{in}}^R \frac{\partial \phi}{\partial t} dr \quad (4.16)$$

By applying the hydrodynamic erosion equation (4.12) and making an appropriate approximation of the integral, we arrive at

$$d\bar{\phi} = \frac{d\Lambda}{\Lambda} (\phi_0 - \bar{\phi}) + \frac{\lambda Q}{2\pi H(r_{in} + \gamma\Lambda)} f(\bar{\phi}) dt \quad (4.17)$$

Where the constant γ ($0 < \gamma < 1$) is a parameter influenced by the (unknown and non-essential to the current formulation) profile of the porosity field. The constant γ originates from an approximate evaluation of the integral $\Lambda^{-1} \int f(\phi) r^{-1} dr \approx (r_{in} + \gamma\Lambda)^{-1} f(\bar{\phi})$ that emerges when equations (4.12) and (4.13) are substituted into equation (4.17). The parameter γ can be determined self-consistently. It is important to note that equation (4.17) can be regarded as the defining equation governing the time evolution of the mean porosity within the presented framework.

Equations (4.9) and (4.10) suggest a correlation between the increments of the plastic zone radius R (and consequently the plastic zone depth Λ) and the mean porosity.

$$d\Lambda = dR = -F_\phi d\bar{\phi} - F_\sigma d\sigma_{out} \quad (4.18)$$

Where the vectors F_ϕ and F_σ are:

$$F_\phi = \frac{\frac{\partial F}{\partial C} C' + \frac{\partial F}{\partial \Phi} \Phi'}{\frac{\partial F}{\partial R}}, \quad F_\sigma = \frac{\frac{\partial F}{\partial \sigma_{out}}}{\frac{\partial F}{\partial R}} \quad (4.19)$$

prime denotes differentiation with respect to mean porosity. By combining equations (4.18) with equation (4.17), we obtain an evolution equation for the plastic zone depth.

$$d\Lambda = u dt \quad (4.20)$$

where u is the speed of advancement of the plastic yielding, and it is defined by:

$$u = -\frac{F_\phi}{1 + \frac{(\phi_0 - \bar{\phi})}{\Lambda} F_\phi} \times \left[\frac{\lambda Q}{2\pi H(r_{in} + \gamma\Lambda)} f(\bar{\phi}) - \frac{(\phi_0 - \bar{\phi})}{\Lambda} F_\sigma \frac{d\sigma_{out}}{dt} \right] - F_\sigma \frac{d\sigma_{out}}{dt} \quad (4.21)$$

Equations (4.18) and (4.21) imply that the mean porosity varies with time according to:

$$d\bar{\phi} = \frac{1}{1 + \frac{(\phi_0 - \bar{\phi})}{\Lambda} F_{\phi}} \times \left[\frac{\lambda Q}{2\pi H(r_{\text{in}} + \gamma\Lambda)} f(\bar{\phi}) dt - \frac{(\phi_0 - \bar{\phi})}{\Lambda} F_{\sigma} d\sigma_{\text{out}} \right] \quad (4.22)$$

Note that due to radial symmetry, σ_{out} may depend on time or be a pure constant. In the latter case, all terms proportional to $d\sigma_{\text{out}}$ vanish. Therefore, we have a system of two ordinary differential equations that explicitly govern the evolution of the plastic zone depth and mean porosity. These can be regarded as the defining equations of the proposed model, which may be considered as a kinematic formulation of the hydro-mechanical description of the erosion process. The produced mass can then be derived, following the approach of presented in Gravanis et al., (2015), to obtain

$$dM = dt \rho_s 2\pi H \lambda \int_{r_{\text{in}}}^{r_{\text{in}} + \Lambda} f(\phi) \frac{Q}{2\pi H r} r dr \approx \rho_s \lambda \Lambda f(\bar{\phi}) Q dt \quad (4.23)$$

where ρ_s is the mass density of the rock. In the last equality, we simplify the integral using its first approximation based on the mean porosity. The parameter γ , introduced in equation (4.17), is determined self-consistently to ensure that the eroded mass at late times is equal to the mass of the sample. This completes the formulation of the framework.

The general analysis of the model can be clarified by introducing appropriate scales for the various variables to facilitate the use of dimensionless quantities. By examining equation (4.17) and in particular the second term, we may define the following time scale relevant to the erosion phenomenon:

$$\frac{1}{T} = \frac{\lambda Q}{2\pi(r_{\text{out}} - r_{\text{in}})H} \quad (4.24)$$

Furthermore, the speed scale is as follows:

$$U = \frac{r_{\text{out}} - r_{\text{in}}}{T} = \frac{\lambda Q}{2\pi H} \quad (4.25)$$

We are now able to specify the hydrodynamical erosion and degradation models. The hydrodynamical equation (4.12) assumes a specific form, subject upon the assumption that the function f is as follows:

$$f(\bar{\phi}) = \left(\frac{\bar{\phi}}{\phi_0} \right)^{a_H} (1 - \bar{\phi}) \quad (4.26)$$

This function involves a constant exponent a_H . It satisfies the property $f(1)=0$ and also drops to zero at low porosity values. The degradation models which are considered take the form:

$$C(\bar{\phi}) = C_0 \left(\frac{1 - \bar{\phi}}{1 - \phi_0} \right)^{a_D} \quad (4.27)$$

$$\Phi(\bar{\phi}) = \Phi_0 \left(\frac{1 - \bar{\phi}}{1 - \phi_0} \right)^{a_D} \quad (4.28)$$

where C_0 and Φ_0 represent the initial values of the cohesion and friction angle, respectively. The exponent a_D quantifies a non-linear dependence of the cohesion/friction angle, indicating that degradation intensifies at lower porosity values. Additionally, the exponent a_H quantifies a power-law dependence of the erosion rate within the lower porosity regime.

For the sake of completeness, we document the initial mass of the sample, which will be utilized in scaling the sand production mass in the next section.

$$M_{\text{sample}} = (1 - \phi_0) \rho_s \pi (r_{\text{out}}^2 - r_{\text{in}}^2) H \quad (4.29)$$

4.3 Analysis and Results

Following the formulation of the proposed model in section (4.2), attention now shifts to analyze the model parameters to serve as sensitivity analysis. The discussion begins with the implementation of the simplified model in the experimental investigation conducted by Papamichos et al., (2001) to serve as the validation of the proposed framework.

Given that the sand production coefficient λ is unknown, the model can solely be evaluated by estimating its values for the various levels of external stress examined in the experiment conducted by Papamichos et al., (2001) and by comparing the predicted sand production curves of the new semi-analytical model with the experimental data (see Chapter 3). As demonstrated in Gravanis et al., (2015) variations in flow rate predominantly corresponds to a rescaling of time within these models. Thus the external stress remains the primary non-trivial factor of influence. In this study, λ is employed as the sole fitting parameter for estimations, contrasting with the three-parameter best-fit approach used in the back-analysis procedures of Gravanis et al., (2015), Sarris et al., (2021) and Gravanis et al., (2022). Preliminary findings of this model were previously presented in Gravanis et al., (2022).

4.3.1 Parametric analysis of degradation laws and hydrodynamic constitutive equations

The geometric characteristics of the hollow cylinder sample and the mechanical properties of the rock are summarized in Table 4.1. The necessary physical parameters were obtained from the work of Papamichos et al., (2001).

Table 4.1 Model properties and input data used in the computations (Kakonitis et al., 2023b)

Variable	Value
Geometric properties	
Hollow cylinder internal radius, r_{in} [m]	0.01
Hollow cylinder external radius, r_{out} [m]	0.1
Cylinder height, H [m]	0.2
Rock properties	
Initial cohesion, C_0 [MPa]	3.7
Initial friction angle, Φ_0 [$^\circ$]	37.4
Initial porosity, ϕ_0 [-]	0.3
Mass density, ρ_s [kg/m ³]	2640
Model Parameters	
Degradation control parameter, a_D [-]	{1, 1.5, 2, 3}
Hydro-dynamic control parameter, a_H [-]	{0, 3}

The external stress values employed in that experiment were $\sigma_{\text{out}} = \{7.5, 8, 9, 10, 11\}$ MPa, with a flow rate $Q = 0.5$ L/min, corresponding to a drawdown pressure of 0.15 MPa, which was maintained constant throughout the duration of the experiment. The elastic properties of the material influence solely the pressure-dependent terms within the equation that determines the plastic zone thickness, Λ (Gravanis et al., 2015) consequently, they are deemed irrelevant in the current formulation. The analysis advances to the impact of various degradation rates, as defined by equations (4.27) and (4.28), together with the influence of different hydrodynamic erosion constitutive equations, as defined by equation (4.26).

Figure 4.3 illustrates the relationship between the friction angle and cohesion as functions of porosity. It is observed that the values of porosity start from the initial porosity considered $\Phi=0.3$. The parameter a_D , which represents the rate of softening, governs the speed at which the friction angle and cohesion approach near-zero values. For instance, when considering the highest value of a_D , both the friction angle and cohesion effectively diminish at a porosity level of 0.8.

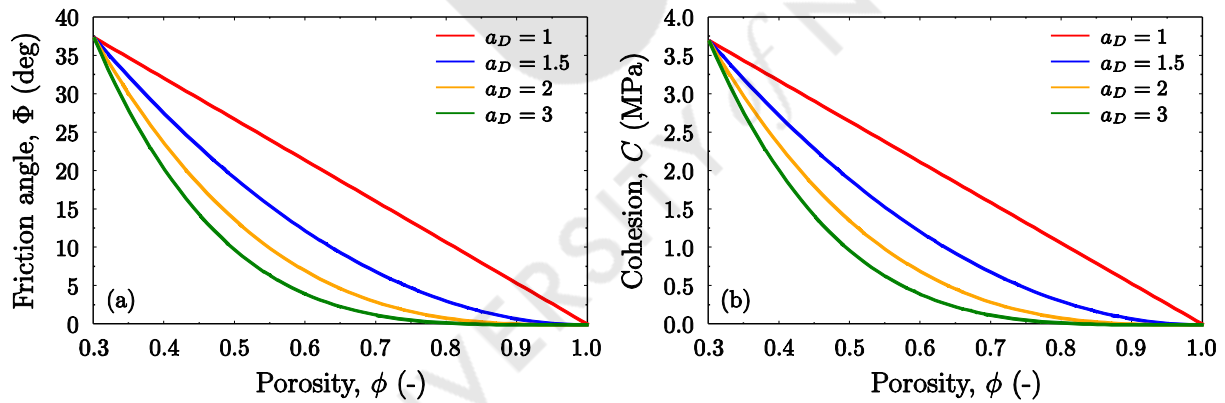


Figure 4.3 Degradation of friction angle and cohesion according to Equations (4-25, 4-26) (Kakonitis et al., 2023b)

Figure 4.4 demonstrates the impact of the degradation control parameter, representing the softening rate a_D , on the progression of (i) sand production, (ii) porosity, (iii) plastic zone depth, and (iv) plastic yielding speed, all expressed in terms of dimensionless time, utilizing the scale defined in equation (4.24).

This analysis pertains to the case where the hydrodynamic control parameter a_H is set to 0. Specifically, sand production is expressed in terms of the dimensionless ratio of eroded

mass/sample mass, and the plastic yielding speed is expressed in the dimensionless form divided by the scale U as defined in Equation (4.25). The plastic zone is given in dimensional form corresponding to the sample size of the experiment conducted by Papamichos et al. (2001). A sensitivity analysis was conducted using the data presented in Table 4.1, with an external stress set at 11 MPa and a flow rate of 0.5 L/sec.

Figure 4.4a demonstrates that the non-linear degradation law exhibits a qualitative difference from the linear law ($a_D=1$) although all cases converge smoothly to a mass ratio value of 1.0. The evolution of porosity is depicted in Figure 4.4b and demonstrates two regimes. Initially, plastic yielding progresses while the erosion process affects the material within the plastic zone. In the second regime, it appears that the plastic zone has already extended to the outer boundary of the rock, thereby activating the erosion process throughout the entire sample.

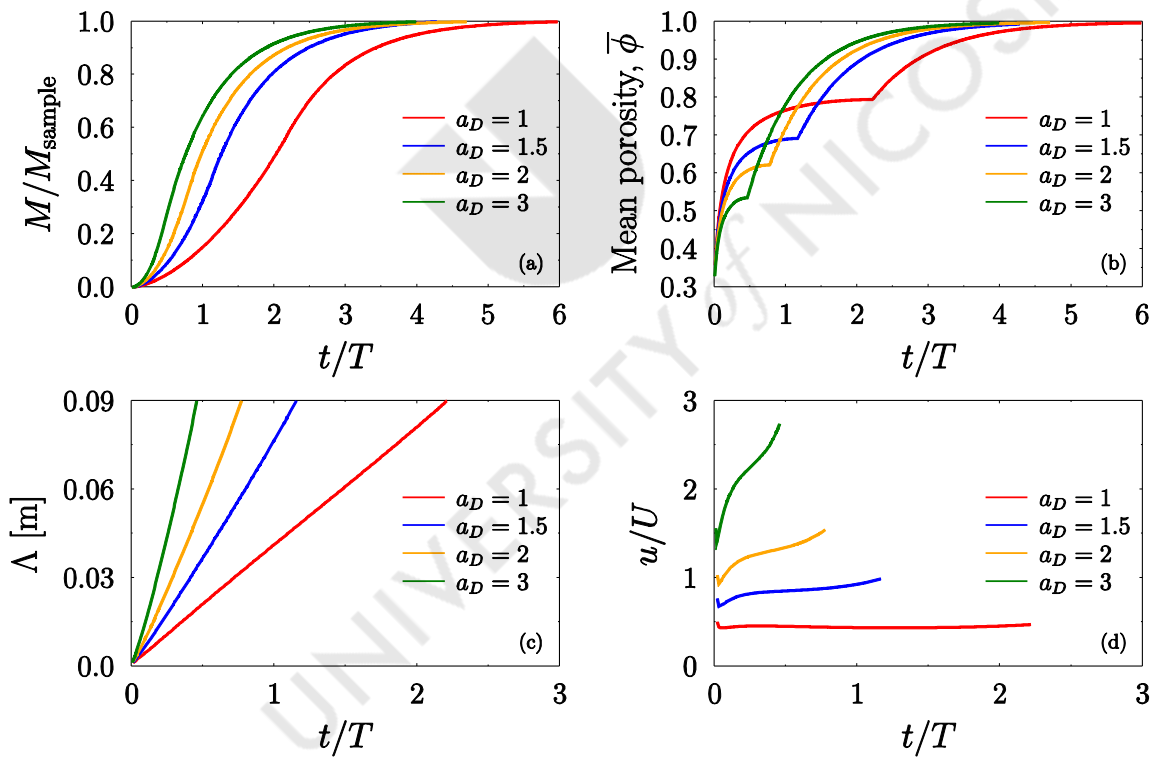


Figure 4.4 Influence of rate of softening (a_D) for exponent $a_n=0$ (constitutive equation 4-24) (Kakonitis et al., 2023b)

It is observed that as the softening rate a_D increases, plastic yielding reaches the outer boundary faster while the mean porosity is initially smaller for higher values of the softening rate, which is primarily a consequence of the larger plastic zone that has been created. Figure 4.4c illustrates the

explicit progression of the plastic zone depth Λ as plastic yielding progresses, considering the radial size of the sample ($r_{\text{out}}-r_{\text{in}}=0.09\text{m}$). Figure 4.4d illustrates the rate of plastic yielding, represented by the slope of the curves in Figure 4.4c, expressed in a dimensionless form. It is observed that this rate increases with the softening rate, beginning from an nearly constant speed for the linear degradation law ($a_D = 1$) to a faster and continuously accelerating yielding as a_D increases.

It is important to note that the case depicted in Figures 4.4(a-d), which pertains to a hydrodynamic exponent of $a_H = 0$ and a linear degradation law with $a_D = 1$ (represented by the red-line curves), corresponds to the conventional forms of the hydrodynamic constitutive and degradation laws employed in the earlier works of Stavropoulou et al., (1998), Papamichos et al., (2001) and Gravanis et al., (2015) that this work extends upon. The case concerning the hydrodynamic exponent $a_H = 3$ of equation (4.24), is illustrated in Figure 4.5. This analysis utilises the same physical variables as the preceding case as ($a_H = 0$). Additionally, the data from Table 4.1 are employed, corresponding to an external stress of 11 MPa and a flow rate of 0.5 L/s.

Figure 4.5a demonstrates once again that the non-linear degradation laws qualitatively differ from the linear law. In this instance, the selected softening rate results in a collapse of all sand production curves. It is also observed that the rate of sand production increases significantly compared to the previous case where $a_H = 0$. This is evident from the considerably reduced time required to erode the entire sample. Accordingly, the modification of the hydrodynamic law in the low porosity regime accelerates the erosion process, and consequently, through degradation, the rate of plastic yielding. This is clarified mathematically that in the low porosity regime, the hydrodynamic law outlined in equation (4.26) assumes the following form:

$$\frac{\partial \phi}{\partial t} = \left(\frac{\phi}{\phi_0} \right)^{a_H} \quad (4.30)$$

This approximation remains valid for early times and is more accurate for small initial porosity. For $a_H \geq 1$, this is integrated to give:

$$a_H = 1 \quad : \quad \phi = \phi_0 \exp \left(\frac{t}{\phi_0} \right) \quad (4.31)$$

$$a_H > 1 \quad : \quad \phi = \phi_0 \left(1 - \frac{a_H - 1}{\phi_0} t \right)^{-\frac{1}{a_H - 1}} \quad (4.32)$$

The range $a_H \geq 1$ encompasses the scenario where $a_H = 3$, as depicted in Figure 4.5. From equations 4.31 and 4.32 it is observed that, for these cases, the porosity increases rapidly in the initial stages of the phenomenon, hence explaining the acceleration of the erosion process for $a_H \geq 1$.

Figure 4.5b illustrates that the transition between regimes occurs within a relatively brief period, that is, the duration until plastic yielding reaches the outer boundary of the sample does not differ significantly. Interestingly, for the case of $a_H = 3$, the porosity curves nearly overlap during the second regime.

The kinematics of the plastic yielding progression are illustrated in Figures 4.5c and 4.5d. Plastic yielding advances at an accelerating rate, which intensifies with higher softening rates denoted by a_D , whereas the speed of advancement attains much higher values as compared to the previously studied scenario where $a_H = 0$.

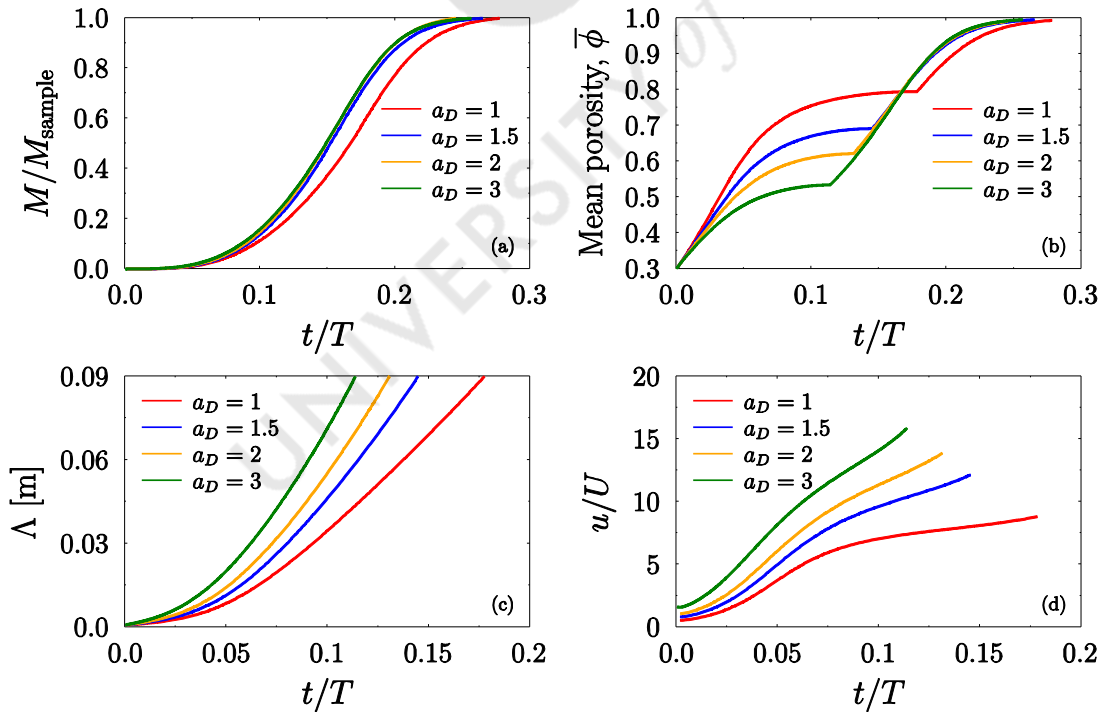


Figure 4.5 Influence of rate of softening (a_D) for exponent $a_H=3$ (constitutive Equation 4-24) (Kakonitis et al., 2023b)

It should be noted that negative values of the exponent a_H result in a concave rather than convex mass production curves, as demonstrated in Figures 4.4a and 4.5a, during the initial stages. This behavior is phenomenologically inconsistent with the experimental data shown in Figure 4.6 in the following. For this reason, only the case $a_H \geq 0$ has been considered.

4.3.2 Comparison with the hollow cylinder experiment of Papamichos et al. 2001

The next step of the analysis is the comparison of the proposed framework with readily available experimentally published data on Hollow cylinder tests, by utilizing the hydrodynamic and degradation constitutive expressions defined in equations (4.26–4.28). The objective of this comparison is to align the theoretically predicted sand production curves derived from the semi-analytical model, with the experimental data reported by Papamichos et al., (2001). Four test cases are evaluated for both the linear ($a_D=1$) and non-linear ($a_D=3$) degradation scenarios, the cases $a_H = 0$ and $a_H = 3$ of the hydrodynamic law were examined. These parameter values correspond to the extreme limits of the exponents analyzed in Section 4.3.1.

Figures 4.6(a-e) present the theoretical sand production curves for the four cases outlined above, compared against the experimental data provided by Papamichos et al., (2001). Additionally, Figure 4.6f displays the estimated values of the sand production coefficient λ (with units m^{-1}), derived from the best fit of the theoretical curves generated by the proposed semi-analytical model to the experimental data across the four cases and respective stress levels. The sand production data from Papamichos et al., (2001) are represented by circle markers, whereas the theoretical curves are distinguished with red corresponding to $a_H=0$ and blue to $a_H=3$. The dashed lines indicate $a_D=1$, and solid lines correspond to $a_D=3$.

This analysis presents the behavior of the considered hydrodynamic and degradation laws in comparison with experimental data across various external stresses. As anticipated, discrepancies are evident between the theoretically predicted sand production curves and the experimental data. Notably, the late stage of experimental sand production is not accurately represented. The observed reduction of the erosion rate may be attributed to the yielded zone, acknowledging that, in the terminal stages, the rock has largely eroded away. Such a picture is qualitatively consistent with our conceptual framework, which describes the process as an interaction between the rate of yielding and erosion progression. However, the simple hydrodynamic and degradation laws

employed are not entirely effective quantitatively. To the best of our knowledge, no existing model adequately captures this behavior to date.

Furthermore, it is possible that the data have been affected by the gradual variation of external stress during the experiment conducted by Papamichos et al., (2001).

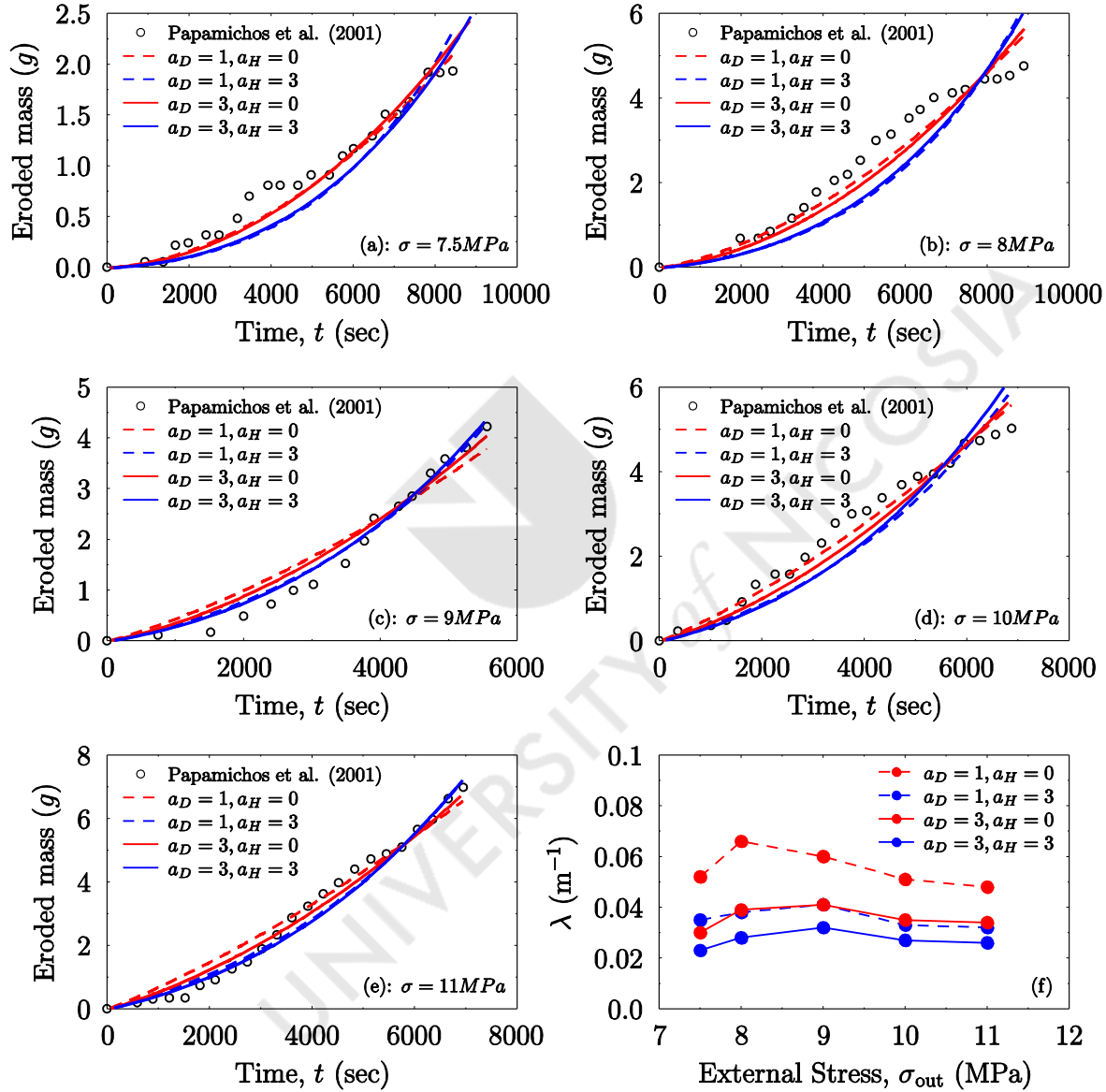


Figure 4.6 Comparison of the test cases of the degradation and hydrodynamic laws (Eqs. 4.24-4.26) against the experiments of (Papamichos et al., 2001). Red color corresponds to $a_H=0$ and blue color to $a_H=3$; dashed lines indicate $a_D=1$ and continuous lines to $a_D=3$. (Kakonitis et al., 2023b)

The semi-analytical framework proposed, which is based on two ODE's may be applied both effectively and efficiently to validate and calibrate more sophisticated hydrodynamic and degradation laws that may more accurately depict actual data.

Additionally, it is noteworthy that the values of the sand production coefficient λ exhibit a certain pattern across all test cases. It is also observed that the variability of λ with respect to stress diminishes as the non-linearity becomes more pronounced in the degradation process and within the low porosity regime of the hydrodynamic laws.

Figures 4.7a and 4.7b illustrate the solution of the semi-analytical model for the simplest scenario associated with hydrodynamic and degradation laws, characterized by parameters $a_H = 0$ and $a_D = 1$. The solution encompasses the two fundamental variables within the kinematic formulation of the erosion process. Namely, the evolution of mean porosity (Figure 4.7a) and the plastic zone (Figure 4.7b) as functions of the external stress. The case where $a_H = 0$ and $a_D = 1$ aligns with the constitutive laws originally employed by Vardoulakis et al., (1996). The solution for the other cases of hydrodynamic and degradation laws demonstrates a qualitative resemblance to the behavior illustrated in Figure 4.7 and are not presented to maintain clarity of the manuscript rather than to confuse the reader with similar cases and repetitive figures. The most straightforward case where $a_H = 0$ and $a_D = 1$ which aligns with the constitutive laws used originally employed by Vardoulakis et al., (1996) and Kakonitis et al., (2023b).

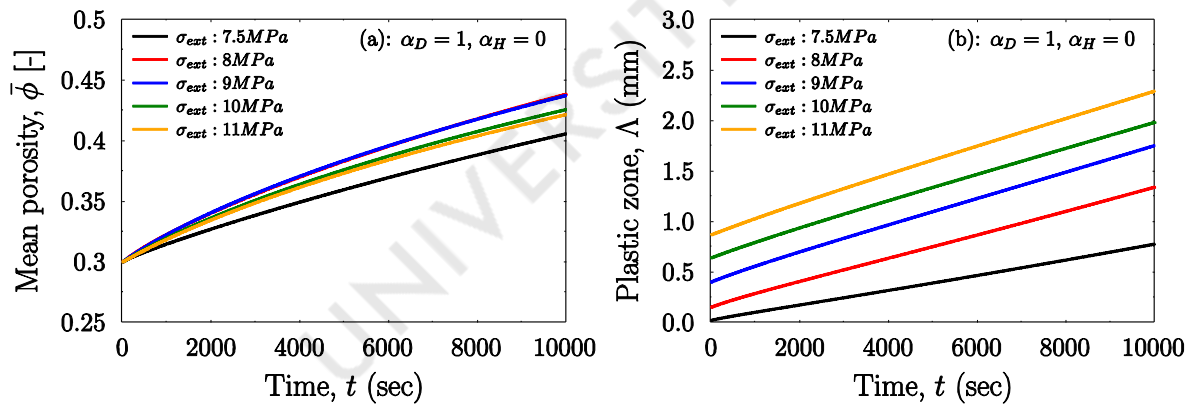


Figure 4.7 Mean porosity and plastic zone evolution for different external stresses (Kakonitis et al., 2023b)

It may be observed that the evolution of the mean porosity does not exhibit an obvious trend with respect to the external stress, primarily due to the effect of the calibrated sand production coefficient. Overall, the predicted mean porosity is not significantly different for the various values

of external stress considered. On the other hand, the plastic zone depth consistently increases with the increase of externally applied stresses, as anticipated. It is worth noting that, for the smallest applied stress (7.5 MPa), which is proximal to the no-yielding threshold (7.41 MPa), the plastic zone depth demonstrates a substantial variation relative to its initial value. This phenomenon is attributable to the significant role played by erosion and material degradation in this scenario.

In our previous study (Kakonitis et al., 2023a), we proposed a power law relationship between the sand production coefficient λ and the external stress, based on the assumption that λ is zero at the minimum stress required for yielding (7.41 MPa). In this chapter we extend the work by introducing a revised power law relationship that incorporates a decreasing exponential factor. In the previous study of chapter 3, a power-law relationship was proposed between the sand production coefficient λ and the external stress, assuming that λ approaches zero at the minimum stress required for yielding (7.41 MPa). In the following part of this research, the formulation is extended by introducing a revised power-law relationship that incorporates a decaying exponential factor.

$$\lambda(\sigma) = \alpha \left(\frac{\sigma - \sigma_{\min}}{\sigma_{\min}} \right)^{1/6} \exp \left[-\beta \left(\frac{\sigma - \sigma_{\min}}{\sigma_{\min}} \right) \right] \quad (4.32)$$

where $\alpha[\text{m}^{-1}]$, $\beta [-]$ are constants to be determined by best fit, $\sigma_{\min}=7.41$ MPa and the power law exponent is taken from (Kakonitis et al., 2023a). Constants α and β after best fit determination are presented in Table 4.2.

Table 4.2 Best fit parameters α , β of the eq. (4.32) (Kakonitis et al., 2023b)

$a_H [-]$	$a_D [-]$	$\alpha[\text{m}^{-1}]$	$\beta [-]$
0	1	0.111	1.598
3	1	0.070	1.446
0	3	0.065	1.124
3	3	0.048	1.041

The parameter values of α and β are presented in Table 4.2, corresponding to the different cases of hydrodynamic and degradation laws. Equation (4.32) is plotted in Figure 4.8 alongside the previously obtained values of λ , which are displayed in Figure 4.6f. It is important to note that the values of the parameters α and β depend on the material.

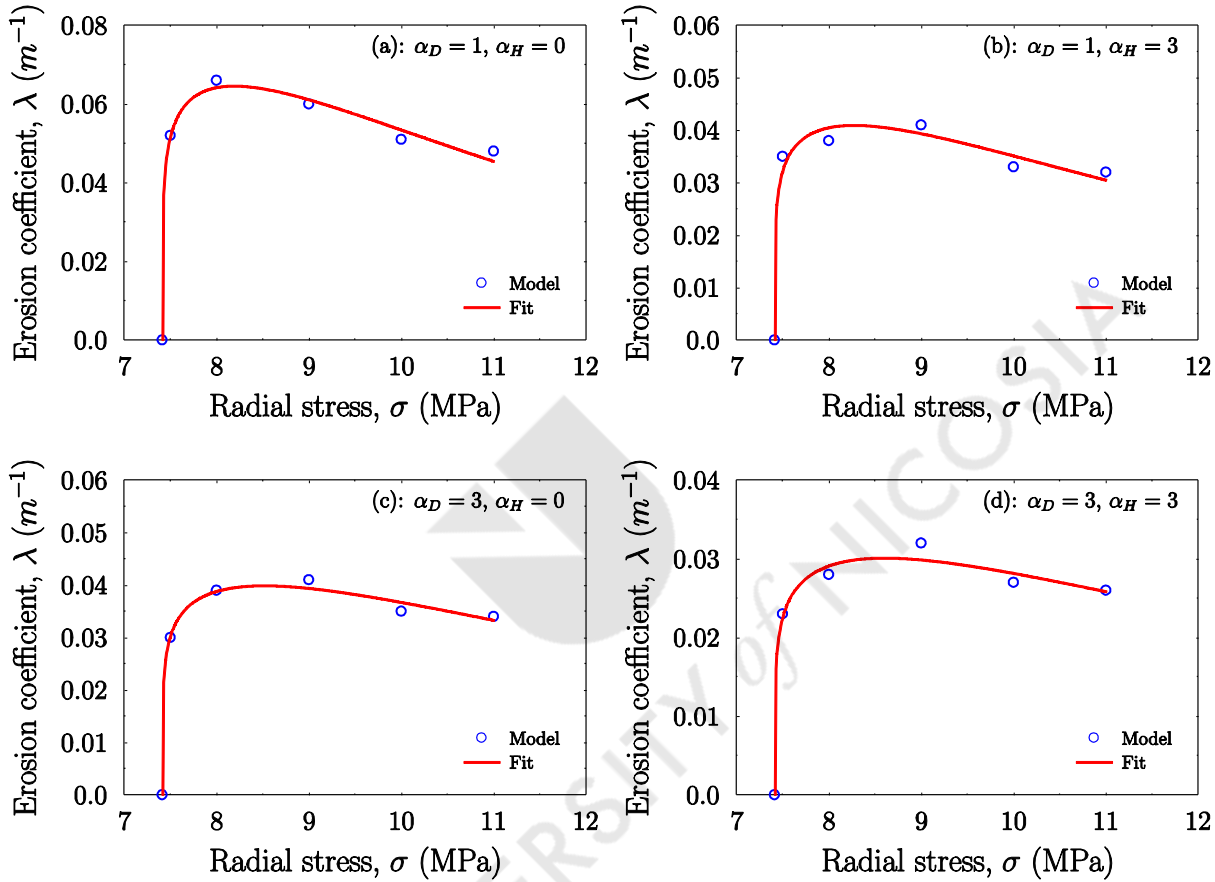


Figure 4.8 Best fit of equation (4-28) for the four cases of degradation and hydrodynamic laws (Kakonitis et al., 2023b)

4.3.3 Comparison with the experiment of Kooijman et al., (1996)

To demonstrate better the capabilities of the proposed model we have compare also with the experiment, as documented in the work of Kooijman et al., (1996) and further referenced in Van den Hoek et al., (2000) that was conducted on an artificial sandstone block. According to rock mechanics standards, this block is classified as a weak rock and measures 26.25 cm by 26.25 cm by 38 cm. The sandstone specimen features a central horizontal hole of 25.4 mm in diameter, aligned with the longer axis of the sample. To analyse the erosion process, a far-field stress was

applied to the surfaces of the sample, with the ratio of effective vertical to horizontal stresses set at $\sigma_v/\sigma_h = 2$. The stresses varied continuously throughout the sanding test, as illustrated in Figure 4.9a, which depicts the radially applied stress. Additionally, the flow rate Q was varied during the experiment, as shown in Figure 4.9b, corresponding to a maximum drawdown pressure of 25 psi, equivalent to 0.17 MPa (Kooijman et al., 1996). At this point it is considered important to note that the comparison between an actual block specimen and our cylindrical hollow cylinder is possible because the stress anisotropy ratio is constant ($\sigma_v/\sigma_h = 2$) mimicking an isotropic stress field and the block dimensions are about 10 times larger than the inner hole of the sample. These two factors make comparison feasible. The relevant parameters to our semi-analytical model, concerning the artificial block sample utilised in the sanding experiments conducted by Kooijman et al., (1996) and Van den Hoek et al., (2000), are summarized in Table 4.3.

In the context of the developed formulation, the experimental conditions were approximated with the following way. (A) The geometry of the artificial sample is approximated as a cylinder with an outer radius of $r_{out}=26.25/2 \text{ cm} = 13.125 \text{ cm}$. The external stress σ_{out} is assumed to be isotropic and equivalent to σ_v as determined in the experimental test conducted by Kooijman et al., (1996), with its time variation illustrated in Figure 4.9a.

Table 4.3 Relevant parameters of the current formulation with the experiment of (Kooijman , et al., 1996)

Variable	Value
Artificial Sample Properties	
Initial cohesion, C_0 [MPa]	1.25
Initial friction angle, Φ_0 [$^\circ$]	37.5
Initial porosity, ϕ_0 [-]	0.34
Mass density, ρ_s [kg/m ³]	1750
Model Parameters	
Degradation control parameter, a_D [-]	1
Hydro-dynamic control parameter, a_H [-]	0
Sand production coefficient, λ [m ⁻¹]	0.020

When approximating σ_{out} in this manner, it is essential for the initiation of sand production, as predicted by the model, to remain consistent with the experimental data. In particular, for the

specified artificial sample, the minimum external stress required to initiate material plastic yielding is determined to be 2.58 MPa. This value is entirely consistent with the onset of erosion observed in the sanding experiment, as will be discussed below.

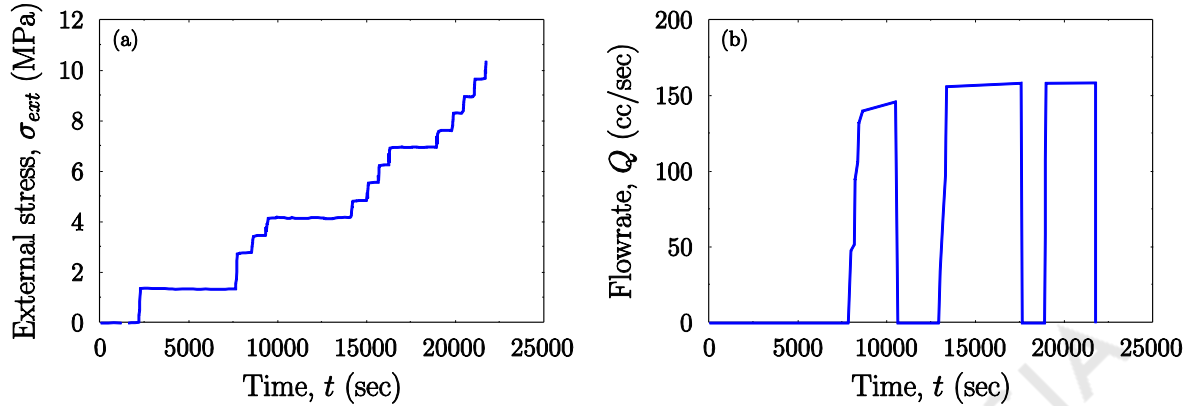


Figure 4.9 Boundary conditions of (a) external stress and (b) flowrate for modelling the sanding experiment of Kooijman et al., (1996)

Figure 4.10 illustrates the predicted sand production mass curve in (gr) obtained through the semi-analytical model, assuming a constant sand production coefficient $\lambda = 0.020 \text{ m}^{-1}$ (refer to Table 4.3), and utilizing the most straightforward hydrodynamic and degradation laws corresponding to parameters $a_H=0$ and $a_D=1$. The sand production coefficient is calibrated as a constant, thereby remaining independent of external stress for simplicity. Future work will focus on calibrating the predefined model $\lambda(\sigma)$, as presented in equation (4.32).

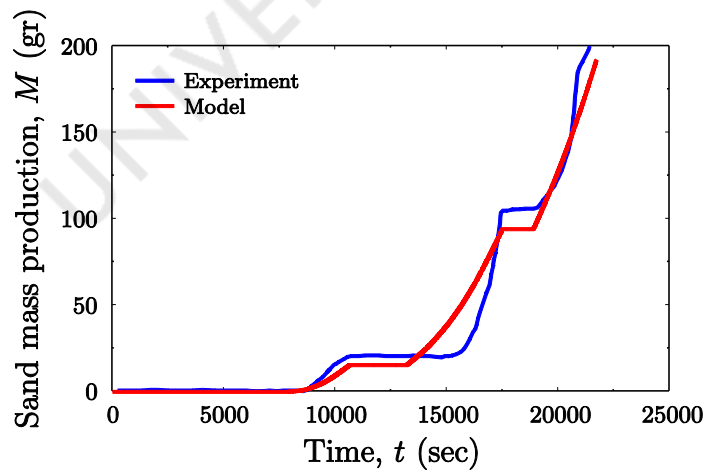


Figure 4.10 Comparison between the proposed semi-analytical model and the experimental data of Kooijman, et al., (1996). See also Van den Hoek et al., (2000) (Kakonitis et al., 2023b)

From examining Figure 4.10, it is observed that the predicted sand production curve depicted with red continuous line aligns quite well with the experimental data shown with blue continuous line. The apparent discontinuities are a direct result of the increasing level of external stress and flow rates considered as boundary conditions. A qualitative distinction between the curves is that the experimental sand production remains relatively constant between 13k to 15k seconds, despite the flow rate not being zero during this interval. This behavior is possibly attributable to the non-continuous nature of the actual erosion process. It should be emphasized that both curves indicate a simultaneous onset of sand production, which is consistent with our model assumption that erosion is directly linked with plastic yielding. The onset of sanding in the experiment occurs when σ_v is approximately 2.6 MPa. As previously stated, this is consistent with the initiation of plastic yielding within the context of our experimental modelling.

Figure 4.11 illustrates the evolution with time of the fundamental variables, namely the mean porosity and plastic zone depth, based on our modeling of the experiment conducted by Kooijman et al., (1996). According to the solution derived from our model, it is observed that periods of erosion advancement alternate with intervals during which erosion ceases.

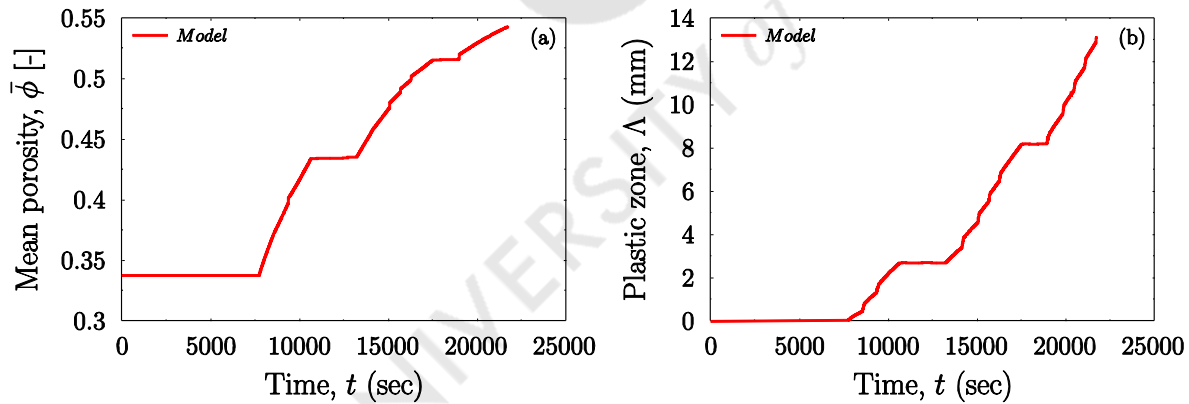


Figure 4.11 Mean porosity and plastic zone evolution for the experimental test of Kooijman et al., (1996) see also Van den Hoek et al., (2000) (Kakonitis et al., 2023b)

In particular, the mean porosity ceases to change when the flow rate diminishes, while the development of the plastic zone depth also halts when the external stress remains unchanged (see also Figures 4.9a, 4.9b that serve as boundary conditions). The trends of both variables are evidently influenced by the increase in the externally applied stress.

4.4 Framework Extension to Account for Variable Boundary Conditions

An extension of the previous formulation was the incorporation of a power-law relationship between the erosion coefficient λ and variable applied external radial stress was introduced. This improvement enables the systematic evaluation of how stress variability influences the sand production curve and hence shed some light to the sand production coefficient λ . This enhancement improves the previous model, which assumed a constant λ , and offers a more accurate foundation for calibrating erosion behavior across various rock types.

4.4.1 Modifications on the Mathematical Formulation

A new mathematical formulation was introduced to simplify the hydrodynamic erosion model to the greatest extent while preserving its ability to capture the key physical mechanisms at play. The mechanisms to be preserved namely are: (1) plastic yielding, (2) erosion, and (3) material degradation. The fundamental physical processes of the initially proposed model are preserved and include the poromechanical behavior of the rock and the sand production process, which commences at the inner surface of a hollow cylindrical rock sample.

The mathematical framework follows the methodology initially established by Vardoulakis et al., (1996) and expanded upon by Papamichos et al., (2001), Gravanis et al., (2015), and lately by Kakonitis et al., (2023a). This research further advances these principles by incorporating a stress-dependent erosion coefficient λ , in the hollow cylinder tests conducted under varying external stress and flow rate as boundary conditions.

The equations governing sand production can be solved by assuming a homogeneous porous solid, assuming that all pores within the rock are filled with a single-phase fluid. Also, in this modified mathematical framework, the principal dynamic variables are (i) the depth of the plastic zone and (ii) the average porosity within the yielded portion of the rock. These variables evolve over time, reducing the problem to a system of two ordinary differential equations. The mechanisms underlying the new updated formulation are based on several key observations:

- a. The depth of the plastic zone Λ can be determined in a quasi-static manner by ensuring stress continuity across the boundary of the plastic zone, thereby minimising the impact of fluid pressure. This assumption remains valid under laboratory conditions, where fluid pressure generally remains two orders of magnitude lower than the externally applied radial boundary stress (Papamichos et al., 2001; Kakonitis et al., 2023b).

- b. The region of the rock that has undergone yielding evolves in accordance with a degradation law that considers the softening of the material. This softening is characterised by the rock's cohesion C and friction angle Φ , both of which are functions of porosity.
- c. The dynamics of the mean porosity within the yielded zone are derived from the hydrodynamic erosion model, as developed in previous works (Vardoulakis et al., 1996; Papamichos et al., 2001; Gravanis et al., 2015, and Kakonitis et al., 2023b). This model describes how fluid flow causes erosion after the rock fails, gradually increasing porosity as the rock degrades in a fully coupled form.

The mathematical formulation is based on the schematic shown in the figure 4.12 below. The rock sample is a hollow cylinder with inner radius r_{in} , outer radius r_{out} , and height H . A uniform external radial stress σ_{ext} is applied at the boundary, however increasing in magnitude over time. Plastic yielding occurs due to this external radial stress, creating a zone of yielded material with a thickness Λ that begins from the inner radius and extends outward (as shown in Figure 4.12). Additionally, the flow rate Q is regarded as a variable boundary condition that changes over time, reflecting the evolving conditions during the test, thus simulating more appropriately the sand production hollow cylinder test.

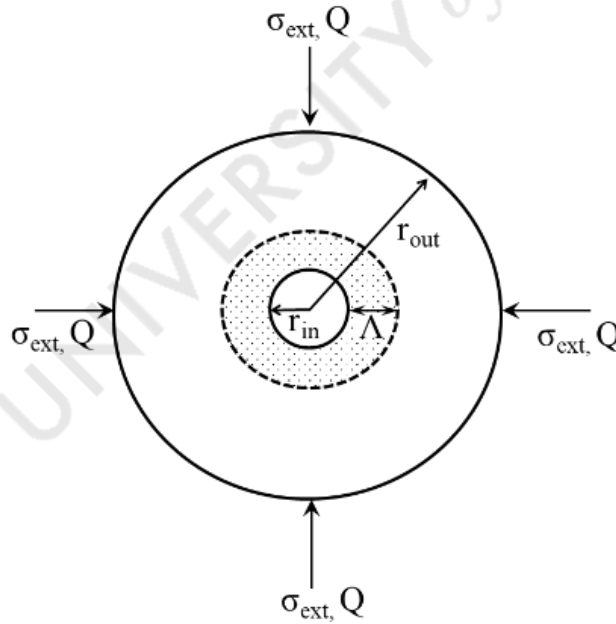


Figure 4.12 Schematic of the hollow cylinder domain and externally applied variable boundary conditions σ_{ext} and Q (Kakonitis et al., 2024).

According to research evolved over time, Papamichos et al., (2001), Gravanis et al., (2015) and Kakonitis et al., (2023b), assumed that sand production occurs solely within the yielded zone of the rock. Although erosion may be affected by the magnitude of strain and potentially extend beyond the plastic zone, this assumption is adopted for simplicity and aligns with the conventional understanding that erosion predominantly results from plastic deformation. By embracing this assumption, the analysis is streamlined without substantially compromising the physical accuracy of the model.

As in the first mathematical formulation presented in this work in section 4.3, the conventional representation of the porosity field, as depicted in the classical Kozeny-Karman equation, is simplified. The mean porosity $\bar{\varphi}$ within the yielded region with thickness Λ will serve as one of the primary dynamic variables. The formulation presumes axisymmetric conditions, consistent with the hollow cylinder experimental setup. It incorporates three principal components: (a) the Mohr-Coulomb criterion for granular materials, (b) a degradation law to model material erosion, and (c) a hydrodynamic law to account for sand grain production.

Following the work of Kakonitis et al., (2023b), the depth of the plastic zone Λ is governed by an algebraic equilibrium constraint that guarantees mechanical balance across the boundary of the plastic zone as:

$$F(R, C, \Phi, \sigma_{ext}) = 0 \quad (4.33)$$

which describes a surface in the space of the variables R, C, Φ, σ_{ext} . The constraint function F is explicitly given by:

$$F(R, C, \Phi, \sigma_{ext}) = (k-1)S_0 r_{in} R^{k+1} + (k+1)S_0 r_{out}^2 r_{in} R^{k-1} - 2(S_0 + (k-1)\sigma_{ext})r_{out}^2 r_{in}^k \quad (4.34)$$

where R is the plastic zone boundary, C is the cohesion of the rock, Φ is the friction angle of the rock, S_0 uniaxial compressive strength and k is a parameter given by:

$$k = \tan^2 \left(\frac{\pi}{4} + \frac{\Phi}{2} \right) \quad (4.35)$$

$$S_0 = 2C\sqrt{k}$$

During erosion, material degradation leads to changes in both the cohesion C , and the friction angle Φ , which subsequently influence the equilibrium constraint, resulting in adjustments to the depth of the plastic zone R . Furthermore, the external stress σ_{ext} , typically exhibits variation over time.

The relationship between the incremental change dR in the plastic zone depth and the variations in C , Φ , and σ_{ext} is expressed as:

$$dF = \frac{\partial F}{\partial R} dR + \frac{\partial F}{\partial C} dC + \frac{\partial F}{\partial \Phi} d\Phi + \frac{\partial F}{\partial \sigma_{\text{ext}}} d\sigma_{\text{ext}} = 0 \quad (4.36)$$

The explicit expression of the partial derivatives in this equation is excessively intricate. Material deterioration is represented through the reliance of cohesion and friction angle on porosity, which in this context, is indicated by the mean porosity within the yielded zone as in the original formulation. This porosity-dependent degradation governs the alterations in mechanical properties as erosion advances.

$$C = C(\bar{\phi}), \quad \Phi = \Phi(\bar{\phi}) \quad (4.37)$$

The erosion law, which originates from filtration theories, is characterised by the constitutive equation presented in the work of Vardoulakis et al., (1996) .

$$\frac{\partial \phi}{\partial t} = \lambda f(\phi)q \quad (4.38)$$

where $\lambda \text{ [m}^{-1}\text{]}$ is the sand production coefficient, q is the Darcy flux of the fluid, and f is a function of porosity, which is taken to be $f(\phi)=1-\phi$. Also, continuity implies that the radial Darcy flux q reads (Gravanis et al., 2015):

$$q = \frac{Q}{2\pi Hr} \quad (4.39)$$

Following previous work (Kakonitis et al., 2023b), the mean porosity could be written as:

$$d\bar{\phi} \approx \frac{d\Lambda}{\Lambda}(\phi_0 - \bar{\phi}) + \frac{\lambda Q}{2\pi H r_{in}} f(\bar{\phi}) dt \quad (4.40)$$

Equations (4.36) and (4.37) specify that the increments of the plastic zone radius R and consequently the plastic zone depth Λ are governed by variations in the mean porosity and external stress. Therefore, changes in the plastic zones are described by:

$$d\Lambda = dR = -F_\phi d\bar{\phi} - F_\sigma d\sigma_{ext} \quad (4.41)$$

where the force vectors are given by:

$$F_\phi = \frac{\frac{\partial F}{\partial C} C' + \frac{\partial F}{\partial \Phi} \Phi'}{\frac{\partial F}{\partial R}}, \quad F_\sigma = \frac{\frac{\partial F}{\partial \sigma_{ext}}}{\frac{\partial F}{\partial R}} \quad (4.42)$$

where prime denotes differentiation with respect to mean porosity. By combining equations (4.40) and (4.41), an evolution equation for the plastic zone depth and mean porosity can be derived.

$$d\Lambda = -\frac{F_\phi}{1 + \frac{(\phi_0 - \bar{\phi})}{\Lambda} F_\phi} \times \left[\frac{\lambda Q}{2\pi H(r_{in} + \gamma\Lambda)} f(\bar{\phi}) dt - \frac{(\phi_0 - \bar{\phi})}{\Lambda} F_\sigma d\sigma_{out} \right] - F_\sigma d\sigma_{out} \quad (4.43)$$

and

$$d\bar{\phi} = \frac{1}{1 + \frac{(\phi_0 - \bar{\phi})}{\Lambda} F_\phi} \times \left[\frac{\lambda Q}{2\pi H(r_{in} + \gamma\Lambda)} f(\bar{\phi}) dt - \frac{(\phi_0 - \bar{\phi})}{\Lambda} F_\sigma d\sigma_{out} \right] \quad (4.44)$$

Due to radial symmetry, the governing equations for erosion dynamics simplify to two coupled ordinary differential equations. These equations establish a kinematic framework for describing the hydro-mechanical behavior of the erosion process. The mass production of sand is then expressed as:

$$dM = \rho_s \lambda \Lambda f(\bar{\phi}) Q dt \quad (4.45)$$

Here, ρ_s denotes the mass density of the rock (Kakonitis et al., 2023b). This completes the formulation of the new updated model.

Within this updated framework, it is assumed that both the cohesion and friction angle decrease linearly as functions of the mean porosity:

$$C(\bar{\phi}) = C_0 \left(\frac{1 - \bar{\phi}}{1 - \phi_0} \right) \quad (4.46)$$

$$\Phi(\bar{\phi}) = \Phi_0 \left(\frac{1 - \bar{\phi}}{1 - \phi_0} \right) \quad (4.47)$$

where C_0 , Φ_0 , denote the initial values of the cohesion and friction angle, respectively.

The sand production coefficient λ is presumed to depend on the external stress, under the condition that λ becomes zero at the minimum stress for yielding, denoted as $\sigma_{\min}$. In accordance with the methodology of (Kakonitis et al., 2023b), a power law dependence expressed as follows has been implemented:

$$\lambda(\sigma) = \alpha \left(\frac{\sigma - \sigma_{\min}}{\sigma_{\min}} \right)^{1/6} \quad (4.48)$$

where $\alpha[m^{-1}]$ is a constant to be determined through the optimal fitting of the experimental data.

4.4.2 Analysis of produced results with the updated formulation

For comparisons, we have used the same experiment from Kooijman et al., (1996) to validate our new formulation. The geometric considerations within our formulation necessitate approximation, given that it is based on cylindrical geometry, while the experiment involves blocks composed of artificial weak sandstone measuring 26.25 cm by 26.25 cm by 38 cm. The tested block incorporates a central hole with a diameter of 25.4 mm, aligned along its major axis. To harmonise the geometric conditions of the sandstone block, we approximate it as a cylinder with an outer radius of $r_{\text{out}} = 13.125 \text{ cm}$, an inner radius of $r_{\text{in}} = 12.7 \text{ mm}$, and a height of $H = 38 \text{ cm}$. These dimensions represent the estimated hollow cylindrical configuration corresponding to the sandstone block. The mechanical properties of the rock are summarised in Table 4.4.

The physical parameters outlined in Table 4.4 were sourced from Van den Hoek et al., (2000), building upon the research conducted by Kooijman et al., (1996). The input values pertaining to the boundary conditions of external stress and variable fluid flow are now modelled as functions of time, rather than as constant values, were the same used in the original formulation and are illustrated in Figure 4.13. The variation of external radial stress over time (measured in MPa) and the cyclic flow rate (measured in cc/sec) reflect the experimental conditions from the sand production tests carried out by Van den Hoek et al., (2000) and Kooijman et al., (1996).

Table 4.4 Model properties and input data for the updated formulation (Kakonitis et al., 2024)

Variable	Value
Artificial Sandstone Properties	
Initial cohesion, C_0 [MPa]	1.25
Initial friction angle, Φ_0 [$^\circ$]	37.5
Initial porosity, ϕ_0 [-]	0.34
Mass density, ρ_s [kg/m ³]	1750
Model Parameters	
Constant Sand production coefficient, λ [m ⁻¹]	0.020
Model eq (4-44) parameter, α [m ⁻¹]	0.0227

To initiate erosion, Van den Hoek et al., (2000) and Kooijman et al., (1996), applied a far-field stress to the sides of the block, maintaining an effective stress ratio of vertical to horizontal stresses equal to $\sigma_v/\sigma_h = 2$. As illustrated in Figure 4.13, the stresses exhibited continuous variation throughout the erosion test, while the flow rate was also adjusted, leading to a maximum drawdown pressure of 0.17 MPa. For the artificial specimen employed in the experiment, the minimum external stress necessary to induce rock yielding was determined to be 2.58 MPa, which aligns well with the onset of erosion observed during this sanding experiment.

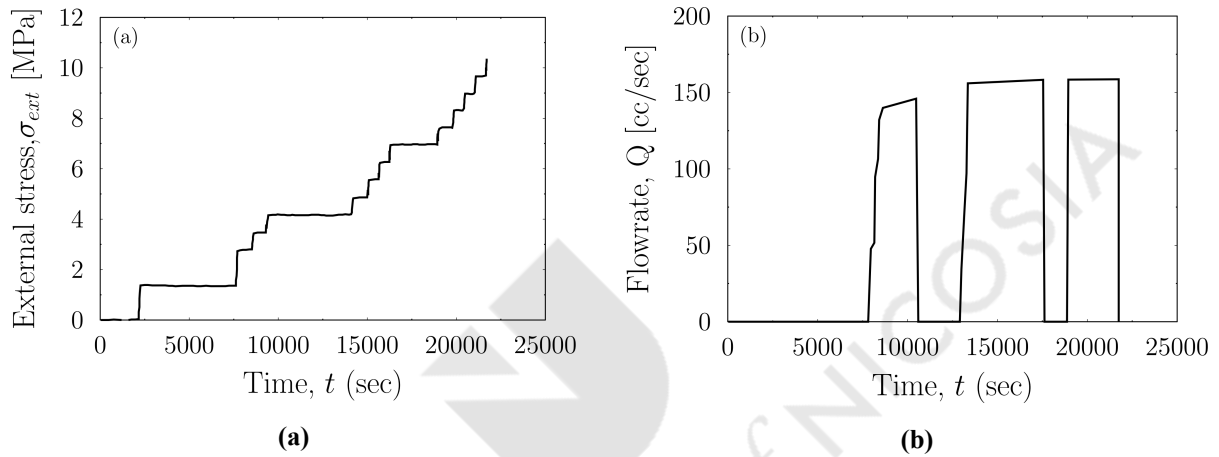


Figure 4.13 Experimental conditions from the sand production test of Van den Hoek et al., (2000), (a) external stress and (b) flow rate as a function of time used as boundary conditions for the analysis.

Figure 4.14 illustrates the cumulative sand production curves derived from this analysis. Specifically Figure 4.14 (a) shows the experimental data obtained from Van den Hoek et al., (2000) and Kooijman et al., (1996), represented by a black solid line. (b) Demonstrates the semi-analytical model developed by Kakonitis et al., (2023b) utilising a constant sand production coefficient $\lambda = 0.020 \text{ m}^{-1}$ (refer to Table 4.1) and linear hydrodynamic and degradation laws, illustrated with a blue solid line. (c) Displays the solution from the new updated formulation, which incorporates a power-law dependence of λ on stress, as shown by the red solid line.

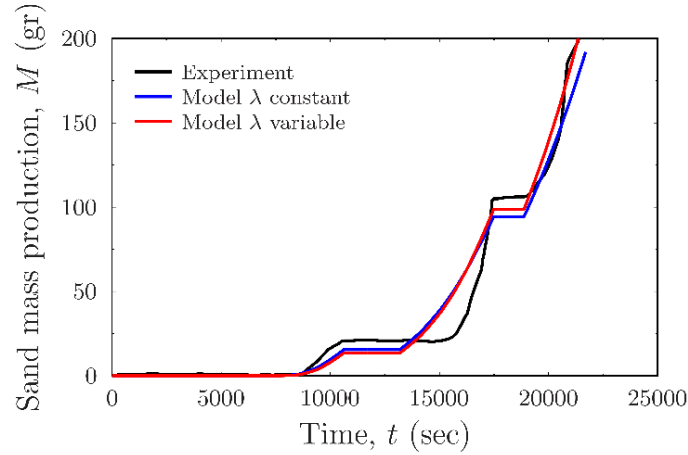


Figure 4.14 Comparison between the cumulative sand production from the experiment of Van den Hoek, et al., (2000) depicted with black line, the model of Kakonitis et al., (2023b) depicted with blue line and the new updated model depicted with red line (Kakonitis et al., 2024).

The comparison of the results is presented in Figure 4.14 and indicate that both cumulative sand production curves, one utilising a constant erosion coefficient λ and the other employing a power law dependence, capture quite well the experimental sand production data. Notably, a slight deviation is observed between the modelling curves and the experimental data within the time interval of 13000 to 15000 seconds, a period during which the flow rate is non-zero. This discrepancy may be attributed to the discontinuous nature of the actual erosion process. A notable feature of the proposed formulation is that both models forecast the initiation of sand production simultaneously with the experimental observations, thereby corroborating our assumption that erosion commences only after the test samples have yielded. In the experiments conducted by Van den Hoek et al., (2000) and Kooijman et al., (1996), erosion initiates when σ_v reaches approximately 2.6 MPa. Furthermore, it is imperative to emphasise that the solution provided by Kakonitis et al. (2023b) closely aligns with the results obtained in this study up to approximately 17.000 seconds. Beyond that point, up to the conclusion of the experiment at 22.000 seconds, the model employing the power-law λ demonstrates marginally superior performance compared to the constant λ model in accurately representing the experimental behavior. It is expected that the disparity between the two models will become increasingly evident over extended periods of sand production.

Figure 4.15 presents a comparison between the solutions provided by Kakonitis et al., (2023b) and the updated model, focusing on the key variables of (a) the evolution of mean porosity (Fig. 4.15a) and (ii) the plastic zone depth (Fig. 4.15b) over time. As depicted in Figures 4.15a and 4.15b, both solutions exhibit a close agreement within the time frame of 0 to 15.000 seconds.

However, beyond 15.000 seconds, the solution derived from the proposed model demonstrates a steepening trend, which can be attributed to the characteristics of the power-law erosion coefficient employed (equation 4.48).

Interestingly, both solutions during the progression of erosion demonstrate alternating periods in which erosion temporarily ceases until the stress level rises and the flow rate recommences (see Figure 4.13 for boundary conditions considered). Specifically, the mean porosity ceases to change when the flow rate declines to zero, whereas the development of the plastic zone depth halts when the external stress remains unchanged (see Figures 4.15a and 4.15b). The behavior of both variables is evidently affected by the increase in the externally applied stress.

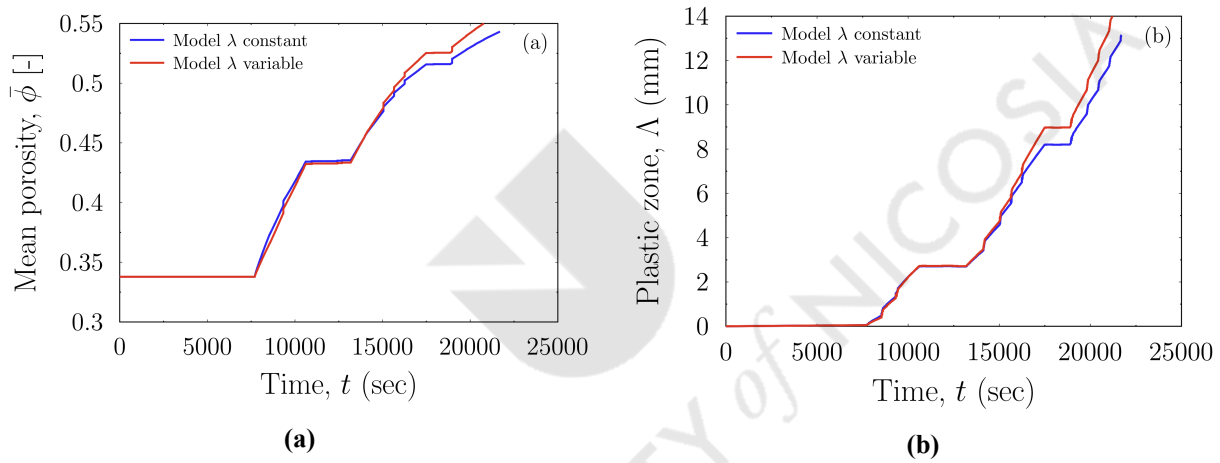


Figure 4.15 Mean porosity and plastic zone evolution for the model of (Kakonitis et al., 2023b) with constant erosion coefficient, $\lambda = 0.02\text{m}^{-1}$ and the proposed model considering power law dependence of λ (eq 16) (Kakonitis et al., 2024).

Finally, Figure 4.16 demonstrates the progression of the erosion coefficient throughout the duration of the experiment. The model employing a constant λ is illustrated by the solid blue line, whereas the model using a power-law λ is represented by the solid red line. This figure emphasises the time dependence of the sand production coefficient λ , as derived from the power-law model specified in equation (4.48), along with the variable boundary condition σ . It is observed that the variable λ curve predominantly remains near the constant value line, exhibiting fluctuations of approximately $\pm 50\%$. Additionally, it initiates at 7.700 seconds when $\sigma = \sigma_{\min}$.

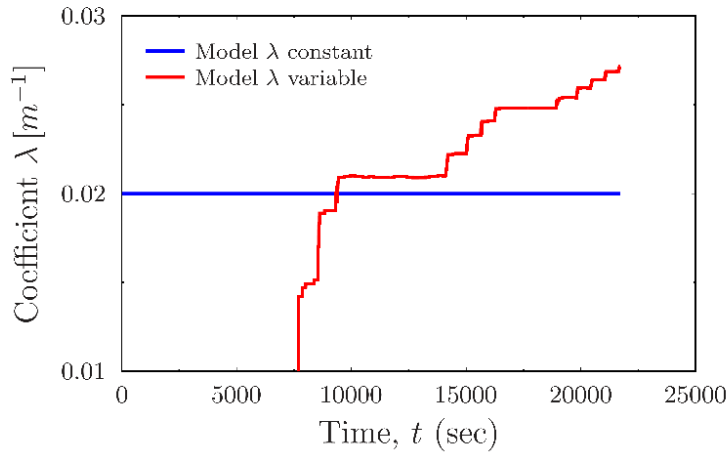


Figure 4.16 Evolution of the erosion coefficient λ with time of the model of Kakonitis et al., (2023b) (continuous blue line) and the updated model considering power law dependence of λ (eq 4.48) (Kakonitis et al., 2024).

4.5 Summary and Conclusions

The novel research presented in this chapter successfully developed, refined, and validated a semi-analytical hydro-mechanical framework for predicting volumetric sand production in hollow cylinder tests. By addressing the inherent complexities of coupled poromechanical failure and fluid-driven erosion, this work provides a robust, computationally efficient alternative to fully numerical simulations. The principal conclusions drawn from the formulation of the model, the integration of a stress-dependent erosion law, and the comparative validation against experimental data are detailed below.

First, the study established that the complex, three-dimensional problem of laboratory sand production can be effectively reduced to a 1D axisymmetric kinematic formulation. This simplification relies on the physically justified premise that erosion is confined entirely within the induced plastic zone. By treating the evolution of this plastic boundary as a quasi-static process governed by a Mohr-Coulomb equilibrium criterion, the framework successfully uncouples the stress field continuity from transient pore pressure effects. This assumption is particularly valid for standard hollow cylinder tests, where the externally applied radial stresses are typically orders of magnitude higher than the generated internal pore pressures. Consequently, the erosion dynamics are precisely captured by a system of two coupled ordinary differential equations governing the mean porosity and the depth of the plastic zone. A central physical insight derived

from this kinematic framework is its ability to replicate the transient nature of the erosion process under variable boundary conditions.

The model mathematically demonstrates that the evolution of the macroscopic variables is strictly dictated by the active loading regime. Specifically, the progression of the mean porosity halts entirely when the fluid flow rate is reduced to zero, reflecting the absolute necessity of a hydrodynamic drag force for particle mobilization. Concurrently, the expansion of the plastic zone depth ceases when the external radial stress remains constant. These alternating intervals of active erosion and stabilization precisely mirror the physical behavior observed during the cyclic loading phases of laboratory sanding experiments.

Building upon the baseline model, a major advancement of this research was the replacement of the conventional, constant sand production coefficient λ with a mathematically rigorous, stress-dependent formulation. Early iterations of the model demonstrated that while a constant coefficient $\lambda=0.020 \text{ m}^{-1}$ could approximate overall trends, but it lacked the physical fidelity required for complex loading paths. To resolve this, an empirical power-law dependency was integrated into the governing equations, linking the magnitude of the erosion coefficient directly to the applied external radial stress. By implementing this specific constitutive relationship, the framework accounts for the non-linear degradation of the rock that stresses may increase beyond the initial yield threshold $\sigma_{\min}$. Analytical results revealed that under variable stress conditions, the dynamic erosion coefficient fluctuates significantly, deviating significantly from the assumed constant baseline value. The superiority of this stress-dependent formulation was confirmed through rigorous comparative analysis against the experimental dataset of van den Hoek et al. (2000).

Both the constant- λ and power-law models successfully predicted the simultaneous onset of sand production at an applied stress of approximately 2.6 MPa, validating the core assumption that macroscopic sanding initiates upon rock yielding. Furthermore, both models demonstrated excellent agreement with the experimental cumulative sand production curve during the initial 15.000 seconds of the test. However, a critical divergence emerged at extended time scales and higher stress magnitudes. Beyond 17.000 seconds, the constant- λ model began to underpredict the volume of produced sand. In contrast, the formulation incorporating the power-law λ accurately captured the accelerated rate of material removal, resulting in a significantly steeper and more accurate cumulative production curve. This confirms that a dynamic, stress-dependent erosion

coefficient is essential for capturing the severe material degradation that occurs at advanced stages of loading.

Ultimately, this semi-analytical framework provides a highly adaptable and practical tool for the systematic calibration of erosion parameters for specific rock types under laboratory conditions. While the current formulation demonstrates exceptional predictive accuracy at the laboratory scale, its application to field-scale scenarios requires further development. Future iterations of this research will focus on integrating the effects of substantial pressure drawdown and accumulated plastic strain. These enhancements will elevate the framework from a laboratory calibration instrument to a comprehensive predictive tool for reservoir management and completion design.

CHAPTER 5- CONCLUSIONS AND RECOMMENDATIONS

5.1 Preamble

This concluding chapter presents a comprehensive synthesis of the research conducted throughout this doctoral research work. It systematically details the primary conclusions drawn from each preceding chapter, integrating the core findings disseminated across the associated peer-reviewed journal publications and conference proceedings that constitute the central contributions of this study. Following this synthesis, the chapter establishes a framework for future research directions aimed at further advancing the hydro-mechanical modeling of sand production.

5.2 Research Work Synthesis

This doctoral research work investigates the complex hydro-mechanical mechanisms governing sand production in weakly consolidated reservoirs during hydrocarbon extraction. By integrating theoretical analysis, advanced numerical simulations, and semi-analytical modeling, the research systematically examines the coupled processes of plastic yielding, hydrodynamic erosion, and material degradation under highly variable stress and fluid flow boundary conditions. The core scientific contributions of this thesis are articulated through a progression of peer-reviewed publications, each addressing specific limitations in existing predictive frameworks to advance the forecasting of sand production.

The initial phase of the research was published in the International Journal of Rock Mechanics and Mining Sciences, that established a high-fidelity finite element model utilizing an Arbitrary Lagrangian-Eulerian (ALE) formulation to simulate laboratory hollow cylinder tests. This numerical approach quantifies the previously under-researched stress dependency of the sand production coefficient (λ), resulting in the derivation of novel empirical expressions that explicitly link λ to the applied external stress and the depth of the induced plastic zone. By demonstrating excellent agreement with experimental datasets at elevated stress levels, this work provides a rigorous mechanical basis for defining erosion onset subsequent to localized plastic yielding.

Building upon these foundational insights, the subsequent phase of the research, published in Geomechanics for Energy and the Environment, develops a novel kinematic semi-analytical framework. This formulation elegantly reduces the complex, spatially distributed erosion

phenomenon into a tractable system of ordinary differential equations governed by two principal state variables: mean porosity and plastic zone depth. By integrating material degradation laws with hydrodynamic erosion criteria, the model provides a computationally efficient and physically transparent predictive tool.

The culmination of this research was presented at the 58th US Rock Mechanics/Geomechanics Symposium (ARMA 2024). This final contribution successfully integrated the stress-dependent erosion coefficient (λ) derived from the numerical analysis into the semi-analytical framework. The comparative validation presented at the symposium confirmed that incorporating a dynamic, power-law dependent erosion coefficient significantly enhances the model's predictive accuracy under variable confining stress conditions, thereby bridging the gap between theoretical rock mechanics and practical sand management challenges.

5.3 Main Conclusions by Chapter

The specific scientific findings and critical insights derived from each phase of this research are detailed below, corresponding to each chapter of this dissertation:

In Chapter 1, that states the problem formulation and hypothesis, we established critical operational and economic imperatives driving sand production research in weakly consolidated reservoirs. The chapter summarizes the central hypothesis of the research work which is the volumetric sand production is fundamentally a coupled hydro-mechanical process where the erosion coefficient (λ) cannot be treated as a static material property. The primary conclusion is that accurate prediction of erosion onset and severity requires λ to be dynamically coupled to the evolving stress state and the mechanical failure of the reservoir rock.

In chapter 2 we present a critical review of current knowledge in sand production mechanics. Through a comprehensive synthesis of the state-of-the-art in sand production modeling, this chapter traces the example shift from purely empirical criteria to advanced coupled hydro-mechanical frameworks. The principal conclusion drawn from this review is the identification of a critical methodological gap which is the persistent reliance on constant erosion parameters within existing volumetric models inherently limits their predictive fidelity under variable field conditions. Furthermore, the review highlights the ongoing computational and calibration challenges associated with complex numerical approaches, establishing the definitive need for the computationally efficient and stress-dependent formulations developed in the subsequent chapters.

In Chapter 3 we investigated the numerical quantification of stress-dependent erosion based on research published in the first journal paper that details the development of a fully coupled finite element model utilizing an Arbitrary Lagrangian-Eulerian (ALE) formulation. The definitive conclusion of this chapter is the quantitative proof of the stress-dependency of the sand production coefficient (λ). By rigorously matching numerical simulations to laboratory hollow cylinder tests, this phase of the research established two novel empirical constitutive laws. (A) the first defines λ as a power-law function of the external radial stress, and (B) the second links it directly to the depth of the induced plastic zone. These formulations successfully establish a mechanical link between the spatial extent of localized rock yielding and the rate of hydrodynamic erosion.

Finally, in Chapter 4 we developed and validated a semi-analytical framework encompassing the findings of the second journal article and the conference proceedings. The main conclusions are the analytical development by formulating, refining, and validating a novel kinematic semi-analytical framework. It mathematically demonstrates that the complex and spatially distributed erosion process can be accurately streamlined into a system of two coupled ordinary differential equations governing mean porosity and plastic zone depth. The culminating conclusion is that integrating the stress-dependent power-law λ , derived in chapter 3, into this framework yields a significant enhancement in predictive accuracy over traditional constant-coefficient assumptions, particularly under high-stress boundary conditions and extended time scales. Validated against multiple independent experimental datasets, including Papamichos et al. (2001), Kooijman et al. (1996), and Van den Hoek et al. (2000), this framework is proven to be a robust and practical tool for calibrating erosion parameters and predicting macroscopic sand production at the laboratory scale.

5.4 Recommendations for Future Research

Building upon the conclusions of this research work, several critical avenues for future research are identified to advance the predictive modeling of sand production.

First, the semi-analytical and numerical models developed herein should be upscaled to simulate field-level conditions by incorporating the reservoir pressure. This work was based on the critical observation that the pore pressure is orders of magnitude less than the confining stress in experimental hollow cylinder tests used for predicting sand erosion masses. Upscaling will indeed make the influence of the pore pressure much greater changing the current frameworks.

Building on the previous recommendation, the influence of transient poro-mechanical phenomena on the initiation and progression of erosion warrants deeper investigation. In field-scale applications, temporal stress redistribution and dynamic pressure transients govern the failure process. Incorporating advanced constitutive relationships, such as strain-dependent friction softening laws, will significantly improve the accuracy of predicting advanced-stage volumetric sand production.

To complement the continuum approaches utilized in this thesis, systematic and standardized calibration protocols for Finite Element models should be established. Formulating a comprehensive geomechanical benchmark for sanding criteria will align micro-scale numerical simulations with macroscopic experimental observations across various geological formations and stress regimes.

Finally, the proposed mathematical frameworks could be integrated directly into reservoir simulation software suites. This integration will enable production engineers to design proactive sand control strategies and optimize completions dynamically. Translating these theoretical and analytical advancements into practical engineering software will ultimately enhance the safety, sustainability, and economic efficiency of hydrocarbon recovery operations.

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APPENDIX: CONTRIBUTIONS OF THIS WORK

Published Paper 1:

Kakonitis, P., Gravanis, E., & Sarris, E. N. (2023). On the stress dependency of sand production coefficient in hydro-dynamical sanding criteria. *International Journal of Rock Mechanics and Mining Sciences*. Vol.169, pp.105443.

Published Paper 2:

Kakonitis, P., Gravanis, E., & Sarris, E. N. (2023). A hydro-mechanical semi-analytical framework for hollow cylinder sanding tests. *Geomechanics for Energy and the Environment*. Vol. 35, pp.100487.

Published Conference Paper 3:

Kakonitis, P., Papaloizou, L., Gravanis, E., & Sarris, E. (2024). Modeling erosion in hollow cylinder tests: the case of low pressure under variable confining stress. In *ARMA US Rock Mechanics/Geomechanics Symposium*. ARMA.